# Purchasing power parity and uncovered interest parity:

# Another look using stable law econometrics

by

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*to my family*

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## ABSTRACT

In this study, we will re-examine the long-run PPP and UIP relationships by using standard cointegration test procedures but with critical values that are appropriate under infinite variance errors. These tests are performed using monthly observations over the period January 1973 – December 1999 for Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden and United Kingdom against the United States.

Finite variance errors are a basic assumption for the distribution theory used to evaluate test statistics for the analysis of cointegration in PPP and UIP. But some recent studies have suggested that many financial variables, such as exchange rate returns, stock market returns, interest rate movements and commodity price movements, may have infinite variance. In this study we estimate the stability indices of the exchange rate, price index, and nominal interest rate series, and find evidence that most of them have an index of stability $\alpha$ less than 2. That is, it appears from the evidence that a stable non-Gaussian model may be more appropriate for these series in our data. Phillips-Perron unit root tests, along with the critical values in Caner (1998), implemented for determining the order of integration of those series generally cannot reject the null of a unit root. The finding of the non-Gaussian stable errors and the unit root in those series provide the motivation for re-doing the cointegration tests for the PPP and UIP relationships.

For the PPP hypothesis, the results obtained by the multivariate likelihood-based cointegration tests demonstrate that while with the normal error assumption the results

show some evidence of supporting the weak-form PPP relationship with the United States, weak-form PPP with the stable error assumption receives stronger support from the data. However, the restrictions for strong-form PPP are rejected. For the UIP hypothesis, the unrestricted cointegration results are consistent and strongly supportive of long-run UIP relationship with the United States under the assumption of stable errors. However, like PPP, the restrictions for strict UIP relationship are rejected.

# CHAPTER 1
# INTRODUCTION

International linkages among markets for goods, capital and foreign exchange play a key role in the process of exchange rate determination. The determination of exchange rates is of crucial importance to an understanding of the links between the domestic and foreign economies in a small open economy. Either in the goods market by assuming adjustment to purchasing power parity or in the capital market by assuming market clearing based on uncovered interest rate parity, the transmission effects from foreign into the domestic economy have frequently been analyzed.

The purchasing power parity (PPP) hypothesis, which is commonly interpreted as the co-movement of the exchange rate and the relative price level of two countries, underlies much of the modern literature on the balance of payments and exchange rate determination. It also has been viewed as a theory of exchange rate determination in its own right. Therefore, PPP is one of the most thoroughly examined topics in international finance and in economics at large. Yet it remains a highly controversial topic, both from the theoretical and empirical perspectives.

The revival of interest in PPP can be attributed to three factors. The first factor is the advent of floating exchange rates in 1973 because the revival of interest in PPP is related to policy, speculation and corporate planning. Thus there has been increasing concern about the misalignment of the exchange rates of major currencies, which is measured by the deviation from PPP on the assumption that the exchange rate consistent with PPP is the long-run equilibrium rate. In addition, some economists (for example, Mckinnon and Ohno (1989)) have advocated the use of PPP as a monetary standard. The second factor is developments in open economy macroeconomics. PPP is assumed to hold continuously in the flexible price monetary model developed by Frenkel (1976), Mussa (1976) and Bilson (1978); however, it is assumed to hold in the long run only in the sticky price monetary model developed by Dornbusch (1976). Moreover, PPP is an important component of the theoretical models of the balance of payments and has some critical implications for international finance such as capital flows, financing and investment decisions and market efficiency. The third factor is the development of cointegration analysis, which provides a statistical representation of long-run relationships.

The uncovered interest parity (UIP) hypothesis claims an *equilibrium relationship* between the market's expected rate of change of the spot exchange rate and the interest rate differential on perfectly substitutable domestic and foreign assets. Under UIP and the assumption of risk neutrality, if the interest rate differential is different from the expected rate of change of the spot exchange rate, agents will move their uncovered funds across financial markets until the difference is eliminated.

The UIP hypothesis is one of the most intensively researched topics in international finance. Several factors have motivated interest in this topic. First of all, the change to floating exchange rates in the 1970s and the deregulation of world capital markets in the 1980s are believed to have resulted in a high degree of integration between exchange and capital markets around the world. A direct test of the degree of integration between financial markets across countries can be done by the testing of UIP. Second, an investigation of interest rate linkages across countries can be done by the UIP testing. Third, the UIP testing constitutes a test of the hypothesis that an unbiased predictor of the expected spot exchange rate is the forward exchange rate, which is a test of market efficiency. Fourth, the development of cointegration analysis motivates the interest in testing international parity conditions, which include the UIP condition. Finally, UIP plays an important role in modeling exchange rate because the UIP condition is the underlying condition for the sticky price monetary model in Dornbusch (1976), Buiter and Miller (1981), and the flexible price monetary model in Mussa (1976), Frenkel (1976) and Bilson (1978).

The development of cointegration analysis has been a factor leading to the interest in both PPP and UIP conditions. The basic idea is that cointegration is an indication of the existence of a long-run relationship between variables. Moreover, cointegration analysis is a technique used in conjunction with time series data.

Finite variance errors are a basic assumption for the distribution theory used to evaluate test statistics for the analysis of cointegration in PPP and UIP. But some recent studies have suggested that many financial variables, such as exchange rate returns, stock market returns, interest rate movements and commodity price movements, may have infinite variance. In this study, we will re-examine the long-run PPP and UIP relationships by using standard cointegration test procedures but with critical values that are appropriate under infinite variance errors. These tests are performed using monthly observations over the period January 1973 – December 1999 for Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden and United Kingdom against the United States.

Chapter 2 has a review of the 'law of one price', strong-form PPP, weak-form PPP, covered interest parity (CIP) and UIP hypotheses. Chapter 3 summarizes previous empirical findings regarding PPP and UIP hypotheses. Chapter 4 describes methodology about the unit root tests, the stable distribution, estimation of stability index, the residual-based cointegration tests and the multivariate cointegration tests. Using the methodology described in chapter 4, the empirical results of PPP and UIP are presented in Chapters 5 and 6, respectively. The summary and conclusions are in Chapter 7.

# CHAPTER 2
# REVIEW OF PPP AND UIP HYPOTHESES

## Purchasing Power Parity Hypothesis

Cassel (1918) coined the term "purchasing power parity" for the concept of a link between exchange rates and national price levels. In particular, reflecting on economic developments during World War I, Cassel wrote:

> The general inflation which has taken place during the war has lowered this purchasing power in all countries, though in a very different degree, and the rates of exchanges should accordingly be expected to deviate from their old parity in proportion to the inflation of each country.
>
> At every moment the real parity between two countries is represented by this quotient between the purchasing power of the money in the one country and the other. I propose to call this parity "the purchasing power parity." As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place, the actual rate of exchange cannot deviate very much from this purchasing power parity. (p. 413)

Absolute PPP hypothesis is that the equilibrium exchange rate at a particular point in time is equal to or determined by relative prices. The condition of absolute PPP is usually derived from the "law of one price" which relates exchange rates to commodity prices. By assuming tradable and homogeneous goods and no impediments to international trade, the "law of one price" holds for each of the goods:

$$P_t^i = S_t P_t^{i^*} \tag{2.1}$$

where $P_t^i$ and $P_t^{i^*}$ denote the prices of good $i$ at home and abroad respectively, and $S_t$ is the exchange rate (home currency price of a unit of foreign currency).

Let $P_t$ and $P_t^*$ denote the price indices for the home and foreign countries, defined according to

$$P_t = \sum_{i=1}^{n} \alpha_i P_t^i \quad \text{and} \quad P_t^* = \sum_{i=1}^{n} \alpha_i P_t^{i^*}$$

where the weights $\alpha_1$, ..., $\alpha_n$ sum to one. Thus, relying on the "law of one price" in the integrated, competitive world market, the absolute PPP will be

(2.2) $$S_t = \frac{P_t}{P_t^*},$$

or be in the logarithmic form as

(2.3) $$s_t = p_t - p_t^*$$

where $s_t = \ln S_t$, $p_t = \ln P_t$, and $p_t^* = \ln P_t^*$. The restrictions necessary for this condition to hold continuously are well known. Homogeneity means that, if prices are multiplied by the same constant, PPP remains unchanged. Symmetry means that $p$ and $p^*$ have opposite coefficients, equal to +1 and −1, respectively.

However, some researchers prefer to go with a less restrictive version of PPP, which simply relies on a real exchange rate being mean-reverting. The real exchange rate $Q$ is defined as the nominal exchange rate adjusted by the relative price:

(2.4) $$Q_t = \frac{S_t P_t^*}{P_t}.$$

This can be expressed in logarithmic form as

(2.5) $$q_t = s_t + p_t^* - p_t,$$

where $q_t = \ln Q_t$. Thus, the alternative way of expressing PPP is to say that the real exchange rate, $q_t = (s_t + p_t^* - p_t)$, should continually equal zero. That is, if the real exchange rate is mean-reverting, a current disturbance to the nominal exchange rate/relative price configuration will eventually be offset. This can be expressed as

(2.6) $$q_t = \rho q_{t-1} + \alpha + \varepsilon_t, \qquad 0 < \rho < 1$$

Recent work has concentrated on the application of cointegration method which focus on equation (2.3) rather than the real exchange rate. Interpreting PPP as an equilibrium condition and allowing for temporary deviations from equilibrium yields

(2.7) $$s_t = p_t - p_t^* + \varepsilon_t$$

where $\varepsilon_t$ is a zero-mean stationary process. It possesses two properties: the homogeneity of degree one and the symmetry condition.

In practice, different countries use different price index weights to calculate the price level, so that the above equation may not be particularly useful. Moreover, absolute PPP may not hold if there are restrictions on trade such as tariffs or quotas, if there are transport costs or if there is imperfect information about prices in the two countries. Therefore, we use weak-form PPP to overcome this problem.

This weak-form PPP, which is a less restrictive version of long-run PPP, may take the form:

(2.8) $$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \varepsilon_t.$$

This form is based on the "law of one price" by allowing for transportation costs, trade impediments, product differentiation and the presence of non-traded goods. According to equation (2.8), $\alpha_1 \neq 1$ or $\alpha_2 \neq 1$ are related to differences in price index weights and for presence of non-traded goods, and $\alpha_0 \neq 0$ is related to existence of transportation costs and trade impediments. The strong-form PPP exists when $\alpha_0 = 0$, $\alpha_1 = 1$ and $\alpha_2 = 1$.

## Interest Parity Hypothesis

It was common knowledge among policymakers by the late nineteenth century that the behavior of exchange rates could be influenced through the adjustment of interest rates. The perception was reinforced as knowledge of the forward exchange market spread through practical banking circles.[1] However, it was not until organized trading in forward exchange expanded rapidly following World War I that Keynes (1923) wove together the first systematic presentation. Now, this theory is referred to as the interest rate parity hypothesis, which has the two main forms – covered interest parity and uncovered interest parity.

### Covered interest parity

The covered interest parity hypothesis originally developed by Keynes (1923) postulates an equilibrium relationship between the spot exchange rate, the forward exchange rate, domestic interest rates and foreign interest rates. The theory stipulates that short-term capital funds have a natural tendency to flow from a low-return financial center to a high-return one, thus equalizing domestic return and covered foreign return. This equilibrium condition hypothesis can be expressed as

[1] Einzig (1970, p214)

$$1 + i_t = \frac{F_{t,k}(1 + i_t^*)}{S_t} \tag{2.9}$$

where $i_t$ and $i_t^*$ are the home and foreign interest rates respectively, $S_t$ is the spot exchange rate and $F_{t,k}$ is the $k$ period forward exchange rate at time $t$ (both exchange rates stated as the home currency price of one unit of foreign currency). After rearrangement, it becomes

$$i_t = i_t^* + \frac{F_{t,k} - S_t}{S_t} \tag{2.10}$$

where $(F_{t,k} - S_t)/S_t$ is interpreted as the forward premium $p_t$, also as 'the cost of covering'. Equation (2.10) would imply[2]

$$\frac{F_{t,k} - S_t}{S_t} \approx f_{t,k} - s_t \approx i_t - i_t^*, \tag{2.11}$$

where $f_{t,k} = \ln F_{t,k}$. The above relationship provides an expression for the forward premium or discount that merchants or investors would have to pay at time t to hedge or "cover" the exchange ate risk associated with a contract to receive or deliver foreign currency at time t+k.

If $i < i^*$, the covered arbitrage funds will flowing through the spot market to the foreign country and this will tend to raise the price of the foreign currency in the sport market, i.e. $S$ rises, while the selling of the forward foreign currency ('to bring the funds home') will reduce the price of the foreign currency in the forward market, i.e. reduce $F$. After rearranging the above equation, we can get the covered interest differential CD:

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[2] Note that, for $1 + i_t^*$ and $1 + i_t$ close to 1, (2.9) implies

$$F_{t,k}/S_t = 1 + (i_t - i_t^*)/(1 + i_t^*) \approx 1 + i_t - i_t^* \text{ and hence } f_t - s_t \approx i_t - i_t^*.$$

(2.12) $$CD_t = i_t - i_t^{*} - p_t$$

If CD>0 there will be capital inflow as the return in the foreign country inclusive of the cost of covering is less than the return in the home country. If CD<0 there will be capital outflow. If CD=0 portfolios are in equilibrium.

**Uncovered interest parity**

The UIP hypothesis associated with Fisher (1930) postulates an equilibrium relationship between the expected rate of change of the spot exchange rate and the short-term interest rate differential on perfectly substitutable financial assets denominated in different currencies. It can be expressed as

(2.13) $$1 + i_t = \frac{(1 + i_t^{*})S_{t+k}^{e}}{S_t}$$

where $S_{t+k}^{e}$ is the spot exchange rate expected to prevail in period t+k. The left hand side is per period return earned investing in the home country financial instrument, and the right hand side is the expected per period return investing in the foreign financial instrument. However, the investor remains uncertain about the exchange rate until the date of conversion arrives: the foreign exchange risk is left uncovered during the interval between time t and time t+k. After rearrangement of equation (2.13), it becomes

(2.14) $$i_t = i_t^{*} + \frac{S_{t+k}^{e} - S_t}{S_t}$$

This is approximately the same

(2.15) $$s_{t+k}^{e} - s_t \approx i_t - i_t^{*}$$

where $s^e_{t+k}$ is the logarithm of exchange rate expected to prevail in period t+k. Thus we define the uncovered interest differential UD:

(2.16) $$UD_t = i_t - i^*_t - (s^e_{t+k} - s_t)$$

If $UD_t>0$ there will be capital inflow to the home country because the expected rate of return on home assets is higher than on foreign assets. If $UD_t<0$ there will be capital outflow from the home country and if $UD_t=0$ asset portfolios are in equilibrium. Interpreting UIP as an equilibrium condition and allowing for temporary deviations from equilibrium yields

(2.17) $$s^e_{t+k} - s_t = i_t - i^*_t + \varepsilon_t$$

where $\varepsilon_t$ is a zero-mean stationary process.

# CHAPTER 3
# PREVIOUS EMPIRICAL TESTS

## Previous Empirical Tests on PPP

Much recent work on determining the validity of PPP has focused on testing equation (2.8). If $s_t$, $p_t$, and $p_t^*$ are unit root processes and if $\varepsilon_t$ is a stationary process, then $s_t$, $p_t$, and $p_t^*$ are cointegrated with cointegrating vector $[1 \quad -1 \quad 1]'$. Thus, assuming that $s_t$, $p_t$, and $p_t^*$ are unit root processes, the following testable conditions are implied by long-run PPP. For both strong-form and weak-form PPP, $s_t$, $p_t$, and $p_t^*$ are cointegrated. That is, there exist constants $\alpha_0$, $\alpha_1$ and $\alpha_2$ such that

(3.1) $$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \varepsilon_t$$

where $\varepsilon_t$ is a stationary process. For strong-form PPP, in addition to the condition of cointegration, the symmetry condition between the domestic and foreign countries, $\alpha_1 = -\alpha_2$ in (3.1), and the long-run proportionality condition between exchange rates and prices, $\alpha_1 = 1 = -\alpha_2$, must hold.

There are different ways for testing cointegration.

### The Engle-Granger cointegration method (1987)

The Engle-Granger cointegration method simply estimates equation (3.1) by ordinary least square (OLS) and applies the Augmented Dickey-Fuller test to the residuals. Let $\{\hat{\varepsilon}_t\}$ be the series of the estimated residuals of the long-run relationship. We can perform an

Augmented Dickey-Fuller test on these residuals to test the order of integration of the $\{\varepsilon_t\}$ sequence, although the critical values of the test statistic must be adjusted because OLS residuals are being used in place of the true $\varepsilon_t^{'}$s. Estimate the autoregression of the residuals:

$$\Delta\hat{\varepsilon}_t = a_1\hat{\varepsilon}_{t-1} + \sum_i a_{i+1}\Delta\hat{\varepsilon}_{t-i} + \varphi_t \tag{3.2}$$

Engle and Granger have tabulated the appropriate critical values for this test when there are two variables. Engle and Yoo (1987) provide critical values if more than two variables appear in the equilibrium relationship.

In the recent exchange rate literature, the Engle-Granger two-step cointegration method has been applied to aggregate price data by Enders (1988), Mark (1990), Patel (1990), and Taylor (1988). Enders (1988), Mark (1990) and Taylor (1988) constrain the coefficient on the relative price terms to be equal and opposite, that is, they impose symmetry, while Patel (1990) estimates equation (3.1) in unconstrained fashion. With relative wholesale price terms constructed for Canada, Germany, and Japan against the United States for the period January 1973 to November 1986, Enders (1988) estimates equation (3.1) and is unable to reject the null of no cointegration in any instance. Taylor (1988) has the same conclusion using a similar data set. Using a number of OECD bilateral rates based on, respectively, the U.S. dollar, U.K. pound, and Japanese yen as the home currency for the period June 1973 to February 1988 (consumer prices), Mark (1990) finds only one instance (out of 13) when the null of no cointegration is rejected. Using quarterly data spanning the period 1974-86 for Canada, Germany, Japan, the Netherlands, and the United States (a variety of bilateral exchange rate combinations are considered for these countries), Patel (1990) reports that the null is rejected in only 4 instances out of a total of 15. Therefore, this group of papers suggests that long-run PPP does not hold.

Even though the Engle and Granger (1987) method is easily implemented, there are some limitations. First, it is possible that the test result is sensitive to which of the variables is chosen to be the dependent variable. Second, as we indicate above, the use of the two-step methodology precludes an actual test of the proportionality and symmetry of the $\alpha$ 's with respect to the exchange rate, although the estimated values are often far from 1 and –1. Third, it relies on a two-step estimator, hence, any error introduced by the researcher in step 1 is carried into step 2. The Johansen (1988) maximum likelihood estimator circumvents the use of two-step estimators and can estimate and test for the presence of multiple cointegrating vectors. Furthermore, the test allows the researcher to test restricted versions of the cointegrating vector(s) and the speed of adjustment parameters.

**The Johansen cointegration method (1988 & 1991)**

Define a vector $x_t$ consisting of n elements (here $x_t = (s_t, p_t, p_t^*)'$ and n=3) and consider a $k$-th order VAR:

(3.3) $$x_t = A_1 x_{t-1} + A_2 x_{t-2} + .... + A_k x_{t-k} + C + \varepsilon_t, \quad t = 1,....,T$$

where the coefficients $A_i$, $i = 1,....,k$ are coefficient matrices, $k$ is the number of lags, $C$ is a constant, and $\varepsilon_t$ is an n-dimensional vector of random disturbances assumed to be identically and independently normally distributed $N_n(0, \Sigma)$. We get the vector error correction form from the above multivariate model

(3.4) $$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + .... + \Gamma_{k-1} \Delta x_{t-k+1} + \pi x_{t-k} + C + \varepsilon_t$$

where $\Delta x_t$ is the first difference of $x_t$, $\Delta x_{t-1}$,...., $\Delta x_{t-k+1}$ are first differences of $x_t$ at lags 1, 2,...., $k$-1, and $\pi$ is the (n x n) matrix $-(I - A_1 - ... - A_k)$.

Assume all variables in $x_t$ are I(1). If there is an error-correction representation of these variables as in (3.4), there is necessarily a linear combination of the I(1) variables that is stationary. Rearranging (3.4) yields

(3.5) $$\pi x_{t-k} = \Delta x_t - \sum \Gamma_i \Delta x_{t-i} - C - \varepsilon_t.$$

Because the right hand side is stationary, $\pi x_{t-k}$ must also be stationary. Thus each row of $\pi$ is a cointegrating vector of $x_t$.

The Johansen cointegration method concerns the number of independent long-term stationary relations among the variables contained in $x_t$ or, equivalently, the rank of the (n x n) matrix $\pi$. If the rank of this matrix is zero, all the series in vector $x_t$ are unit-root processes and there is no long-run cointegrating relationship. If the rank of $\pi$ is equal to n, the vector $x_t$ is stationary. If the rank of $\pi$ is equal to $r$, $0 < r < n$, then $x_t$ is cointegrated with cointegrating rank $r$.

Following Enders (1995), the Johansen method uses two test statistics to determine the cointegrating rank:

(3.6) $$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i)$$

and

(3.7) $$\lambda_{max}(r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1})$$

where $T$ is the number of usable observations and $\hat{\lambda}_i$ are the characteristic roots of the estimated $\pi$ matrix in descending order.

The $\lambda_{trace}$ statistic tests the null hypothesis that the number of cointegrating vectors is less than or equal to $r$ against a general alternative. When all estimated characteristic roots $\hat{\lambda}_i = 0$, $\lambda_{trace}$ equals zero. Thus, the further the $\hat{\lambda}_i$ is away from zero, the larger the $\lambda_{trace}$. The $\lambda_{\max}$ statistic tests the null that the number of cointegrating vectors is $r$ against the alternative of $r$+1 cointegrating vectors. Similarly, $\lambda_{\max}$ will be small when $\hat{\lambda}_{r+1}$ is close to zero. The null hypothesis is rejected if the value of the statistic in (3.6) or in (3.7) is greater than the corresponding critical value. Johansen and Juselius (1990) calculated the critical values of $\lambda_{trace}$ and $\lambda_{\max}$ in their Monte Carlo analysis, which were refined by Osterwald-Lenum (1992).

Cheung and Lai (1993), Kugler and Lenz (1993), and MacDonald (1993) use the Johansen cointegration method to test for the number of cointegrating vectors among relative prices and exchange rates. Macdonald (1993), and Cheung and Lai (1993) use bilateral U.S. dollar exchange rates while Kugler and Lenz (1993), and MacDonald (1993) use German mark bilateral dollar rates.

With consumer price indices (CPIs) and wholesale price indices (WPIs) constructed for United Kingdom, France, Germany, Switzerland and Canada against the United States for the period January 1974 to December 1989, Cheung and Lai (1993) performed the Johansen test in the VAR(8) framework. The hypothesis of no cointegrating vector ($r$=0) can be rejected at the 5 percent level in all cases, which indicates that the series in $x_t$ are cointegrated, as suggested by long-run PPP.

Using monthly data covering the recent flexible exchange rate period of the DM vis à vis 15 currencies[3], Kugler and Lenz (1993) find that PPP seems to hold ($r$=2 or $r$=1) in the long run for six European currencies: the Pound, Lira, Norwegian Krone, Schilling, Escudo and Peseta, while PPP has to be rejected for the United States and the Canadian Dollar as well as for the Belgian Franc and the Danish Krone.

MacDonald (1993) uses bilateral U.S. dollar exchange rates of the Canadian dollar, French Franc, German mark, Japanese yen and U.K. pound with a wholesale price index and a consumer price index for the period January 1974 to June 1990 to test for a long-run PPP relationship and also to test for the proportionality of the exchange rate with respect to relative prices. He implements multivariate cointegration tests by using a twelfth-order lag in the underlying VAR, that is $k$=12 in (2.3). All but two of the country/price combinations have evidence of at least one unique cointegrating vecotor ($r \geq 1$). For the Canadian dollar and the Japanese yen there is no cointegration, when consumer prices are used but not when wholesale prices are used. On the results, he demonstrates that there is a long-run relationship between a number of bilateral U.S. dollar exchange rates and their corresponding relative prices, but the proportionality of the exchange rate to relative prices does not receive support from the data.

[3] Those 15 currencies include the Swiss Franc, French Franc, Lira, Pound Sterling, U.S. Dollar, Yen, Austrian Schilling, Dutch Guilder, Belgian Franc, Spanish Peseta, Swedish krone, Danish Krone, Canadian Dollar, Portuguese Escudo and Norwegian Krone.

**Panel data method**

Recently, a number of researchers have turned to panel data methods in an attempt to find more evidence of long-run PPP in current floating exchange rate data. A standard panel framework is:

$$s_{it} = \alpha_i + \beta'(p_{it} - p_{it}^{*}) + \{\sum_i \gamma_i D_i\} + \{\sum_t \delta_t D_t\} + \varepsilon_{it}, \tag{3.8}$$

where $i$ indicates that the data has a cross sectional dimension (running from 1 to N). $D_i$ and $D_t$ denote country-specific and time-specific fixed effect dummy variables, respectively. The more recent panel exchange rate literature tests for the stationary of the residual series in equation (3.8) or reparameterizes this equation into an expression for the real exchange rate[4] and tests the panel unit root properties of real exchange rates.

A rapidly growing literature has been inspired by the work of Levin and Lin (1992), who showed that, in situations where there is not enough time series variation to produce good power in unit root tests, a relatively small amount of cross-section variation can result in substantial improvement. The Levin and Lin approach involves testing the null hypothesis that each individual series is I(1) against the alternative that all of the series as a panel are stationary. This approach allows for a range of individual-specific effects and also for cross sectional dependence by the subtraction of cross sectional time dummies.

Panel data methods would appear to be useful but the empirical evidence has been mixed. Lothian (1994), Frankel and Rose (1995) find evidence of mean reversion, rejecting the unit root hypothesis. However, with a sample of four exchange rates against the dollar,

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[4] The real exchange rate, in logarithmic form, is ($s_t + p_t^{*} - p_t$).

Hakkio (1984) cannot reject the random walk hypothesis. Using monthly data from ten industrialized countries, Abuaf and Jorion (1990) find only weak evidence against the unit root hypothesis. Using annual data for a larger sample of industrialized countries, Frankel and Rose (1995) can only reject the unit root null if they impose a homogenous intercept across countries.

However, some literature with panel methods reports much stronger evidence of rejecting the unit root null for real exchange rates during the post Bretton Woods period. By updating the Abuaf and Jorion (1990) data, Jorion and Sweeney (1996) find more evidence against unit roots. Using monthly, quarterly and annual data on WPI and CPI, Wu (1996) find strong evidence against the unit root null. Pedroni (1995) is able to reject the more general hypothesis of no cointegration between the nominal exchange rate and the price differential for both annual and monthly data during the current float. Using data from tradable sectors, Oh (1996) finds strong evidence against the unit root hypothesis with panel data method.

### Previous Empirical Tests on UIP

With the joint assumption of risk neutrality and rational expectations

(3.9) $$s_{t+k} = s^e_{t+k} + e_{t+k},$$

we test the UIP of equation (2.17) $s^e_{t+k} - s_t = i_t - i^*_t + \varepsilon_t$ by estimating

(3.10) $$s_{t+k} = \alpha_0 + \alpha_1 s_t + \alpha_2 (i - i^*)_t + u_{t+k}$$

or

(3.11) $$\Delta s_{t+k} = \beta_0 + \beta_1(i - i^*)_t + v_{t+k},$$

where $u_{t+k}$ and $v_{t+k}$ are error terms in equations (3.10) and (3.11), respectively, and $\Delta s_{t+k} = s_{t+k} - s_t$. Due to the joint assumption of rational expectations and risk neutrality, the restrictions ($\alpha_0$, $\alpha_1$, $\alpha_2$)=(0, 1, -1) and ($\beta_0$, $\beta_1$)=(0, 1) must be satisfied. Also, $u_{t+k}$ and $v_{t+k}$ must be stochastic processes, orthogonal to past information. However, regarding the constant term, non-zero values, i.e. $\alpha_0 \neq 0$ and $\beta_0 \neq 0$, may still be consistent with UIP. Relaxing the assumption of risk neutrality, the constant term may reflect a constant exchange risk premium demanded by investors on foreign versus domestic assets.

We group previous research according to test methods.

**Conventional regression analysis**

This method simply estimates equation (3.10) or (3.11) by ordinary least square (OLS), and performs the tests of coefficient restrictions and error orthogonality on it.

For sterling's effective exchange rates over the period 1972:7-1980:2, Hacche and Townend (1981) estimated equation (3.10) and rejected the restriction error term was orthogonal to past information, even though the results are supportive of the UIP constraints on $\alpha_0$ and $\alpha_1$.

Using Eurocurrency rates denominated in five currencies, Gaab *et al.* (1986) tested the restriction in equation (3.11). They distinguish three forms of UIP – strong form, semi strong form and weak form. If the restriction ($\beta_0, \beta_1$)=(0,1) and E($v_{t+k}v_{t+k-i}$)=0 for $i \geq k$ are not rejected, strong-form UIP is valid. On the other hand, the restriction $\beta_1 > \frac{1}{2}$ is required for semi-strong-form UIP, while even the inequality restriction is relaxed for weak-form UIP. They showed that the weak-form UIP was not rejected in any cases, while the strong-form and semi-strong-form UIP are rejected in all cases. Moreover, the estimates of $\beta_1$ were negative in almost all cases. In addition, they found the absence of conditional heteroscedasticity by regressing $v_{t+k}^2$ on the corresponding interest rate differential and its square. This result is in contrast with the results showed by Cumby and Obstfeld (1984).

Cumby and Obstfeld (1981) showed indirect evidence of a time-varying risk premium by finding serial correlation in $v_{t+k}$, by using weekly data on Eurocurrency rates denominated in six major currencies over the period 1974-80 and the Box-Pierce (1970) and log likelihood tests.

Using panel data on Eurocurrency rats over the period 1975:1-1990:10, Mayfield and Murphy (1992) also showed evidence of rejecting UIP, although they found that allowing for a time-varying risk premium could explain these results.

Loopesko (1984) used daily data on overnight Eurocurrency deposit rates for six countries over various periods falling between 1975:1 and 1978:11 to test the error orthogonality property. He tested if $v_{t+k}$ is determined by its lagged values, lagged values of spot exchange rates and lagged values of total net purchase by domestic and foreign authorities against the foreign currency and also found the rejection of UIP.

Following the procedure performed by Cumby and Obsteld (1981), Tronzano (1988) rejected UIP for Italian lira over the period 1973-86, while Khor and Rojas-Suarez (1991) rejected UIP for the Mexican peso over the period 1987-89.

**Vector autoregression analysis**

Without imposing restrictions, vector autoregression analysis is widely accepted as a tool for forcasting. Following Ito (1988), consider a VAR system:

$$x_t = d + \sum_{j=1}^{m} A_j x_{t-j} + \varepsilon_t \,, \tag{3.12}$$

where $x_t \equiv (s_t, i_t, i_t^*)'$ is a vector of endogenous variables, $d \equiv (d_s, d_d, d_y)$ is a deterministic (constant) vector, $\varepsilon_t$ is a vector of white noise, and $\text{cov}(\varepsilon_t, x_{t-j}) = 0$ for $j \geq 1$. Also they define the uncovered interest parity as:

$$H_0 : E_t s_{t+k} = s_t + i_t - i_t^* \,. \tag{3.13}$$

The null hypothesis can be represented as cross-equational restrictions on the VAR system by using the expected future spot rate (for example the 3-step ahead forecast, i.e. k=3):

$$E_t x_{t+3} = d + A_1 E_t x_{t+2} + A_2 E_t x_{t+1} + \sum_{j=1}^{m-2} A_{j+2} x_{t-j+1} \,, \tag{3.14}$$

where m is the lag length, and $A_u$=[0] for $u > m$.

Taylor (1987) tested UIP by implementing the VAR methodology for six- and twelve-month Eurodeposit rates denominated in seven currencies over the period 1979:7-1986:12 and found evidence favorable to UIP in only two cases (German mark-US dollar and Italian lira-pound). Ito (1988) tested UIP for Japan and found evidence that UIP did not hold for the

Euroyen rate over the period 1973-77 but did hold well over the period of free capital mobility 1981-85.

**Cointegration analysis**

Using the Johansen technique, Karfakis and Parikh (1994) tested the hypothesis for three exchange rates (the British pound, German mark and Japanese yen) with risk premia proxied by the sample variances of domestic and foreign bond yields.[5] They have monthly data for the interest differential, expected and actual bilateral exchange rates and variances in bond yields of dollar and foreign-denominated bonds over the period 1974:1-1988:12 for the U.S.-U.K. and 1974:1-1989:12 for the Germany-U.S. and Japan-U.S. bilateral exchange rates respectively. Because the coefficient $\gamma_1$ is unity in all cases, rational expectations hypothesis can not be rejected. Moreover, the null hypothesis that the coefficient $\gamma_2$ is unity may be rejected on the basis of the observed magnitudes of coefficients.

However, results found by Ngama (1994) for the currencies tested by Karfakis and Parikh (1994) were favourable to UIP in all cases except Japan. Ngama (1994) employed the Phillips-Hansen (1990) estimation method to test the UIP hypothesis with a different model specification. In their model, they have the interest parity forward rate as an explanatory

[5] Karfakis and parikh (1994) estimated the equation

$$s_t = \gamma_1 s_{t+1} + \gamma_2 (i^* - i)_t + \gamma_3 VB_t + \gamma_4 VB_t^* + \upsilon_t$$

such that $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 > 0$, $\gamma_4 < 0$.

variable, and incorporate the variance of the expected rate of change of the spot exchange rate generated by a GARCH-in-mean model.[6]

With the Engle-Granger (1987) and Phillips-Ouliaris (1990) tests, Bhatti and Moosa (1995) re-examined the UIP hypothesis as a long-run equilibrium. They used an alternative model specification[7] and quarterly data on three-month Treasury bill rates denominated in eleven currencies[8] *vis-à-vis* the dollar over the period 1972:1-1993:3. Because cointegration appeared in all cases, their results were consistent and strongly supportive of UIP. In addition, the null hypothesis that the constant term $\alpha = 0$ and the coefficient of interest parity forward rate $\beta = 1$ can not be rejected in any case as judged by the West corrected t statistics.

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[6] Ngama (1994) tested UIP by estimating the equation

$$s_{t+k} = \alpha + \beta\left[s_t\left(\frac{i-i^*}{1+i^*}\right)\right] + \gamma\left({}_k h_{t+1}^{\Delta s}\right) + \varsigma_{t+k}$$

such that (α,β,γ)=(0,1,0).

[7] Bhatti and Moosa (1995) tested UIP by estimating the equation

$$s_{t+1} = \alpha + \beta \bar{f}_t + w_{t+1}$$

where $\bar{f}$ is the interest parity forward rate, such that (α,β)=(0,1).

[8] The currencies include the British pound, Canadian dollar, Japanese yen, Swiss franc, Australian dollar, Swedish kroner, Deutsche mark, Dutch guilder, Belgian franc, French franc, and Italian lira.

# CHAPTER 4
# ECONOMETRIC METHODOLOGY

In recent years, a number of approaches for estimating and testing cointegration relationships have been developed, e.g., Engle and Granger (1987), Phillips and Ouliaris (1990), Phillips and Hansen (1990), Johansen (1988, 1991), Johansen and Juselius (1990), and Hansen (1992). The residual-based $\hat{Z}_\rho$ and $\hat{Z}_t$ tests of Phillips and Ouliaris (1990) and the likelihood-based trace and maximum eigenvalue statistics of Johansen (1988, 1991) are the most commonly used tests for cointegration. Finite variance errors are a basic assumption for the distribution theory used to evaluate these test statistics. But some recent studies have suggested that many financial variables may have infinite variance. For example, exchange rate and stock market return data appear to exhibit extreme outlier behavior (Mandelbrot (1963) and Boothe-Glassman (1987)). Koedijk and Kool (1992) investigate the empirical distribution of black-market exchange rate returns for seven East European currencies focusing on the tails of the distribution. Their results support the existence of finite second moments in exchange rates for only four of the seven countries. Akgiray, et al (1988) examines black-market exchange rates for twelve Latin American countries. Strong evidence is found to support the infinite variance hypothesis. From these results, it may be prudent to allow for infinite variance processes in the economic analysis of exchange rate series[9].

[9] Stable parameters have been estimated for stock returns by Fama (1965), Leitch and Paulson (1975), Arad (1980), McCulloch (1994), Buckle (1995), and Manegna and Stanley (1995); for interest rate movements by Roll (1970), and McCulloch (1985); for foreign exchange rate changes by Bagshaw and Humpage (1987), So (1987a, b), Liu and Brorsen (1995), and Brousseau and Czarnecki (1993); for commodities price movements

Caner (1998) derives the limit laws for the $\hat{Z}_\rho$ and $\hat{Z}_t$ test statistics of Phillips and Ouliaris (1990) and the trace and maximum eigenvalue test statistics of Johansen (1988, 1991) under the assumption that the component processes are symmetric stable processes with identical stability indices. The limit laws consist of functionals of symmetric stable laws and, in the case of Johansen (1991), involve the quadratic variation of a symmetric stable process. They depend on the index of stability $\alpha$ and the number of variables in the system. By calculating the size distortion induced by mistakenly using the conventional critical values of Phillips and Ouliaris (1990) and Johansen (1988, 1991), Caner finds that the size distortion biases the tests towards the hypothesis of cointegration. The critical values that apply under the assumption of weakly dependent errors with infinite variance are provided in Caner (1998) for the cointegration tests. That is, we can construct the Phillips-Ouliaris test statistics or the Johansen test statistics, then use Caner's critical values instead of conventional critical values for testing cointegration.

In this study, we will re-examine the long-run PPP and UIP relationship using monthly observations over the period January 1973 – October 1999 for Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain Sweden, and the United Kingdom with the United States as the foreign country.

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by Dusak (1973), and Cornew, Town and Crowson (1984); and for real estate returns by Young and Graff (1995).

**Unit Root Tests**

Before implementing multivariate cointegration tests, we would like to investigate the orders of integration of the series. In the Gaussian case, standard Augmented Dickey-Fuller statistics are used for unit root tests. Phillips and Perron (1988) developed a modification of the Dickey-Fuller procedure, which allows the disturbances to be weakly dependent and heterogeneously distributed instead of the Dickey-Fuller assumptions of independence and homogeneity. Caner (1998) extended the results in Phillips and Perron (1988) to the case of infinite variance errors. In this study, we implement the Phillips and Perron (1988) procedure with Caner's critical values instead of its conventional critical values to perform unit root tests. We consider the two least-squares regression equations:

(4.1) $$y_t = \mu + \alpha y_{t-1} + u_t,$$

and

(4.2) $$y_t = \mu + \beta(t - \frac{1}{2}T) + \alpha y_{t-1} + u_t,$$

where T is the number of observations, and ( $\hat{\mu}, \hat{\alpha}$ ) and ( $\tilde{\mu}, \tilde{\beta}, \tilde{\alpha}$ ) are the conventional least-squares regression coefficients in equations (4.1) and (4.2), respectively. Let $t_{\hat{\alpha}}$ and $t_{\tilde{\alpha}}$ be the usual t-test statistics for the null hypotheses $\alpha = 1$ in (4.1) and (4.2), respectively. The Phillips-Perron statistics are

(4.3) $$Z(t_{\hat{\alpha}}) = (\hat{S} / \hat{\sigma}_{T\omega})t_{\hat{\alpha}} - (1/2\hat{\sigma}_{T\omega})(\hat{\sigma}^2_{T\omega} - \hat{S}^2)[T^{-2}\sum(y_{t-1} - Y_{-1})^2]^{-1/2},$$

and

(4.4) $$Z(t_{\tilde{\alpha}}) = (\tilde{S} / \tilde{\sigma}_{T\omega})t_{\tilde{\alpha}} - (T^3 / 4\sqrt{3}D_x^{1/2}\tilde{\sigma}_{T\omega})(\tilde{\sigma}^2_{T\omega} - \tilde{S}^2),$$

where $D_X = \det(X'X)$, $\sigma^2_{T\omega} = T^{-1}\sum_{1}^{T} u_t^2 + 2T^{-1}\sum_{s=1}^{w}\sum_{t=s+w}^{T} u_t u_{t-s}$, $S$ is the standard error of the regression, ω is the number of estimated autocorrelations, and $Y_{-1} = T^{-1}\sum_{t=1}^{T} y_{t-1}$.

## Estimation of Stability Index

Stable distributions are a rich class of distributions that allow skewness and heavy tails. The general stable distribution is described by four parameters: an index of stability or characteristic exponent $\alpha \in (0,2]$, a skewness parameter $\beta \in [-1,1]$, a scale parameter $\sigma > 0$ and a location parameter $\mu \in R$.

Following Samorodnitsky and Taqqu (1994), a random variable X has a stable distribution, $X \sim S_\alpha(\sigma,\beta,\mu)$, if X has the characteristic function

$$(4.5) \qquad E\exp(itX) = \begin{cases} \exp\{-\sigma^\alpha |t|^\alpha [1 - i\beta(sign(t)\tan(\frac{\pi\alpha}{2}))] + i\mu t\} & \alpha \neq 1 \\ \exp\{-\sigma |t| [1 + i\beta\frac{2}{\pi}(sign(t))\ln|t|] + i\mu t\} & \alpha = 1. \end{cases}$$

The parameters σ, β and μ are unique (β is irrelevant when α=2).

The above characteristic function becomes $E\exp(itX) = \exp\{-\sigma^2 t^2 + i\mu t\}$ when α=2. That is the characteristic function of a Gaussian (or Normal) random variable with mean μ and variance $2\sigma^2$. One property of non-Gaussian stable distribution is that not all moments exist. Let $X \sim S_\alpha(\sigma,\beta,\mu)$ with $0<\alpha<2$ then

$$E|X|^p < \infty \text{ for any } 0 < p < \alpha$$
$$E|X|^p = \infty \text{ for any } p \geq \alpha$$

where $E|X|^p = \int_{-\infty}^{\infty} |x|^p f(x)dx$ and p is any real number. Thus, the second moment (i.e. variance) of a non-Gaussian stable distribution is infinite.

In order to get clearer picture of the differences between normal distributions and stable distributions, two series with 100 observations were drawn independently from standard normal distribution $N(0,1)$ and $\alpha$-stable distribution with $\alpha$=1.5, respectively. Figure 4.1 shows that the series drawn from $\alpha$-stable distribution is more volatile than the one drawn from $N(0,1)$. Also, from the same distributions, two series with 10000 observations were drawn for calculating the empirical densities shown in Figure 4.2. According to this figure, the empirical density of an $\alpha$-stable distribution has a fat tail and outlier behavior.

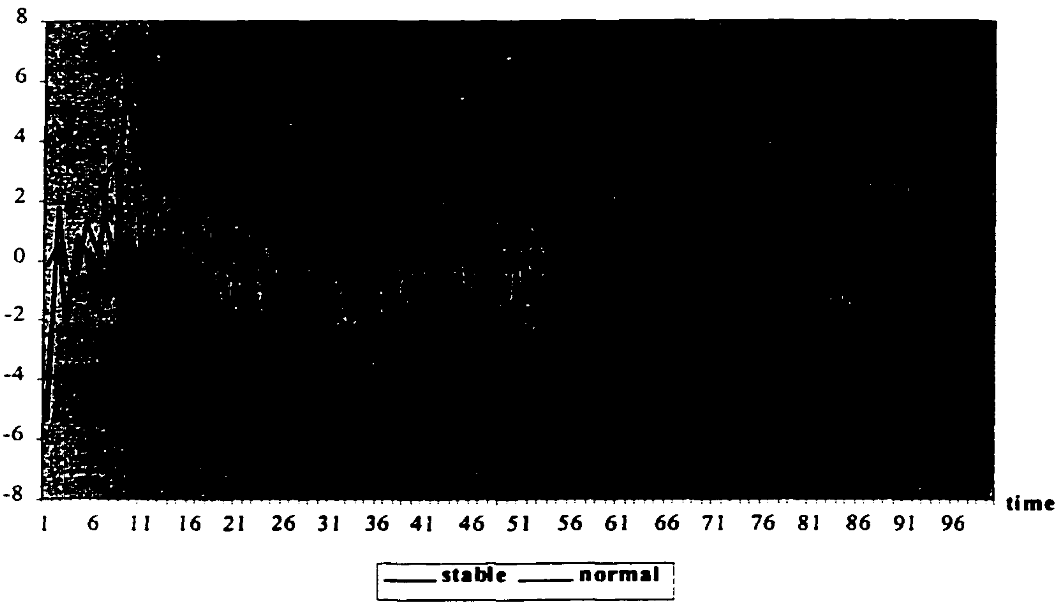



Figure 4.1. Two time series drawn independently from $N(0,1)$ and $\alpha$-stable distribution with $\alpha$=1.5 respectively.

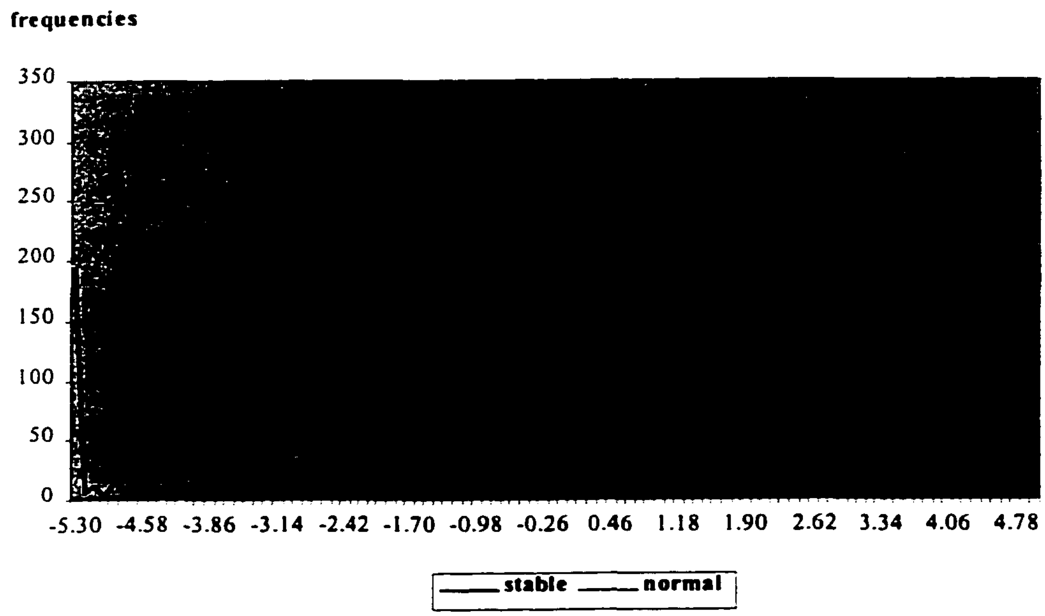



Figure 4.2. Empirical densities of $N(0,1)$ and α-stable distribution with α=1.5

**Maximum likelihood estimation**

Let the parameter vector be denoted by $\vec{\theta} = (\alpha, \beta, \sigma, \mu)$ and the density denoted by $f(x \mid \vec{\theta})$. $\Theta = (0,2] \times [-1,1] \times (0,\infty) \times (-\infty,\infty)$ is the parameter space. For an i.i.d. stable sample $X_1, \ldots, X_n$, the log likelihood function is

$$\ell(\vec{\theta}) = \prod_{i=1}^{n} \log f(X_i \mid \vec{\theta}). \tag{4.6}$$

DuMouchel (1971) and (1973) show that the ML estimator follows the standard theory when $\vec{\theta}$ is in the interior of the parameter space Θ. Therefore, it is consistent and asymptotically normal with mean $\vec{\theta}$ and covariance matrix $n^{-1}\Gamma$, where Γ is the inverse of the 4x4 Fisher information matrix I. The entries of I are given by

(4.7) $$I_{i,j} = \int_{-\infty}^{\infty} \frac{\partial f}{\partial \theta_i} \frac{\partial f}{\partial \theta_j} \frac{1}{f} dx.$$

He also shows that when $\vec{\theta}$ is on the boundary of the parameter space, i.e. α=2 or β=±1, the asymptotic normal distribution for the estimators tends to a degenerate distribution at the boundary point and the ML estimators are super-efficient. By the general theory, away from the boundary of Θ, large sample confidence intervals for each of the parameters are

(4.8) $$\hat{\theta}_i \pm z_{\phi/2} \frac{\sigma_{\hat{\theta}_i}}{\sqrt{n}},$$

where $\sigma_{\hat{\theta}_1},\dots,\sigma_{\hat{\theta}_4}$ are the square roots of the diagonal entries of Γ, $(1-\frac{\phi}{2})$ is the percentage of the confidence interval, and $z_{\phi/2}$ corresponds to the tabular value of standard normal distribution, that comes closet to the specified percentile $\phi/2$.

All but a few stable distributions (Gaussian, Cauchy and Lévy) do not have closed formulas for densities and distribution functions. In practice, this has been a major problem for using stable distributions. In Nolan (1997), the program STABLE gives reliable computations of stable densities for values of α>0.25 and any β, σ and μ. This program now also includes routines for maximum likelihood estimation of stable parameters.

In this study, we use maximum likelihood estimation for stable parameter estimation. As stated above, an i.i.d. data series is required for this method. Thus we need to check if the data series is i.i.d. before estimating the stable parameters.

**Model identification and diagnostic checking**

"Box-Jenkins" is now the standard time series modeling technique for Gaussian time series. Model identification, parameter estimation, and diagnostic checking are the three stages of this modeling technique. Adler, Feldman, and Gallagher (1998) claim that, in principle, the standard Gaussian Box-Jenkins (1976) techniques do carry over to the stable setting. However, in practice, a great deal of care needs to be exercised.

Consider fitting data $\{X_1, X_2,\ldots, X_n\}$ to a linear ARMA(p,q) time sereies model

(4.9) $$X_t - \phi_1 X_{t-1} - \ldots - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \ldots + \theta_q Z_{t-q},$$

with i.i.d. innovations $\{Z_t\}$, which are normal or stable. The identification of lag parameters $p$ and $q$ in the Gaussian case is based on analysis of the sample autocorrelation function (ACF)

(4.10) $$\hat{\rho}(h) = \sum_{t=1}^{n-h} X_t X_{t+h} / \sum_{t=1}^{n} X_t^2, \quad h=1, 2, \ldots,$$

or its mean-corrected version,

(4.11) $$\tilde{\rho}(h) = \sum_{t=1}^{n-h} (X_t - \overline{X})(X_{t+h} - \overline{X}) / \sum_{t=1}^{n} (X_t - \overline{X})^2,$$

where $\overline{X} = n^{-1}(X_1 + \ldots + X_n)$, and of the sample partial autocorrelation function (PACF).

Adler, Feldman, and Gallagher (1998) show that in the α-stable case the limiting distribution of the ACF is given by

(4.12) $$(n/\ln n)^{1/\alpha}(\tilde{\rho}(h) - \rho(h)) \Rightarrow (1 + 2\sum_{j=1}^{q} |\rho(j)|^{\alpha})^{1/\alpha} U/V, \quad h > q,$$

where $\rho(h)$ is an analogue of the population ACF function in the α-stable case, U and V are independent stable random variables. So the above results can be used to plot confidence intervals for the ACF function and identify the parameter $q$ when the distribution of $U/V$ is known.

Note that the distribution of U/V is the limiting distribution of the left hand side of (4.12) when $q=0$. The distribution of U/V can be computed via simulation of numerical integration of the joint density of the vector (U, V) over an appropriate region. Adler, Feldman and Gallagher (1998) found the 97.5% quantiles of U/V, a symmetric distribution, for α < 2 via simulation of 500,000 values of U/V using the S-plus routine for generating stable random variables. They are shown in table 4.1.

In simulation studies, Adler, Feldman, and Gallagher (1998) applied Box-Jenkins procedures for model identification, using three different distributions to construct confidence intervals for the ACF parameters: a stable distribution with the correct α, a Cauchy distribution (α=1) and the Gaussian distribution (α=2). They found that the Cauchy-based limits tend to give the best results.

## Residual-based Cointegration Tests

Let $X_t$ be a vector consisting of p unit root variables with stable errors. Then partition $X_t = (X_{1t,} X_{2t}^{'})'$ into the scalar $X_{1t}$ and (p-1) vector $X_{2t}$. Consider the system of regressions equations:

(4.13) $$X_{1t} = \beta X_{2t} + u_t$$

(4.14) $$u_t = \rho u_{t-1} + \varepsilon_t ,$$

where the $\varepsilon_t^{'}$s are white noise. The null hypothesis in these tests is that the $u_t$ sequence has a unit root, i.e., $\rho = 1$ in (4.14). Residual-based cointegration tests test the null by examing the behavior of the OLS residuals from the fitted version of (4.13).

In this study, we implement the Phillips and Ouliaris (1990) procedure for performing the residual-based cointegration test. Since Caner (1998) extends the Phillips and Ouliaris (1990) procedure from the finite variance errors to the infinite variance errors, we also perform the residual-based test using Caner's critical values instead of its conventional critical values.

**Table 4.1. 97.5% quantiles of U/V**

| α | 97.5% quantile | α | 97.5% quantile |
|---|---|---|---|
| 0.3 | 9.338E+04 | 1.3 | 3.865E+00 |
| 0.4 | 4.625E+06 | 1.4 | 2.814E+00 |
| 0.5 | 7.375E+02 | 1.5 | 2.059E+00 |
| 0.6 | 1.986E+02 | 1.6 | 1.516E+00 |
| 0.7 | 7.745E+01 | 1.7 | 1.096E+00 |
| 0.8 | 3.710E+01 | 1.75 | 9.280E-01 |
| 0.9 | 2.072E+01 | 1.8 | 7.637E-01 |
| 1.0 | 1.240E+01 | 1.9 | 4.765E-01 |
| 1.1 | 8.088E+00 | 2 | 1.960E+00 |
| 1.2 | 5.532E+00 | | |

**Multivariate Cointegration Tests**

In this study, we also implement likelihood-based tests for cointegration. As we stated in chapter 3, we need to consider the Error-Correction form of the VAR model as in equation (3.3):

$$\Delta x_t = \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \pi x_{t-k} + C + \xi_t , \tag{4.15}$$

where the $\xi_t$'s are three-dimensional vectors and satisfy the assumption of being *i.i.d.*, symmetrically distributed and in the normal domain of attraction of a tri-variate symmetric stable law with the same stability index $\alpha$, for all components of the vector, $0<\alpha<2$. This assumption is different from that in equation (3.3), which assumes $\varepsilon_t$ is a three-dimensional vector of identically and independently normally distributed random disturbances.

As we mentioned in chapter 3, the matrix $\pi$ captures the long-run relationship between the p variables in vector X. There are three possibilities for it. First, the rank of $\pi$ is equal to p. It implies that the vector process X is stationary. Second, the rank of $\pi$ is equal to zero, that is, $\pi$ is the null matrix, thus $\Delta X$ is stationary, but the components of X are not cointegrated. Third, the rank of $\pi$ is equal to r, which is less than p. In this case, there are r linear combinations (i.e. r cointegrating vectors) of X that are stationary.

We will compare the multivariate cointegration test statistics, maximum eigenvalue and trace statistics, using Caner's critical values and using the conventional critical values[10].

[10] Asymptotic critical values for the trace and the maximum eigenvalue statistics for a number of endogenous variables up to 11 are given in Osterwald-Lenum (1992).

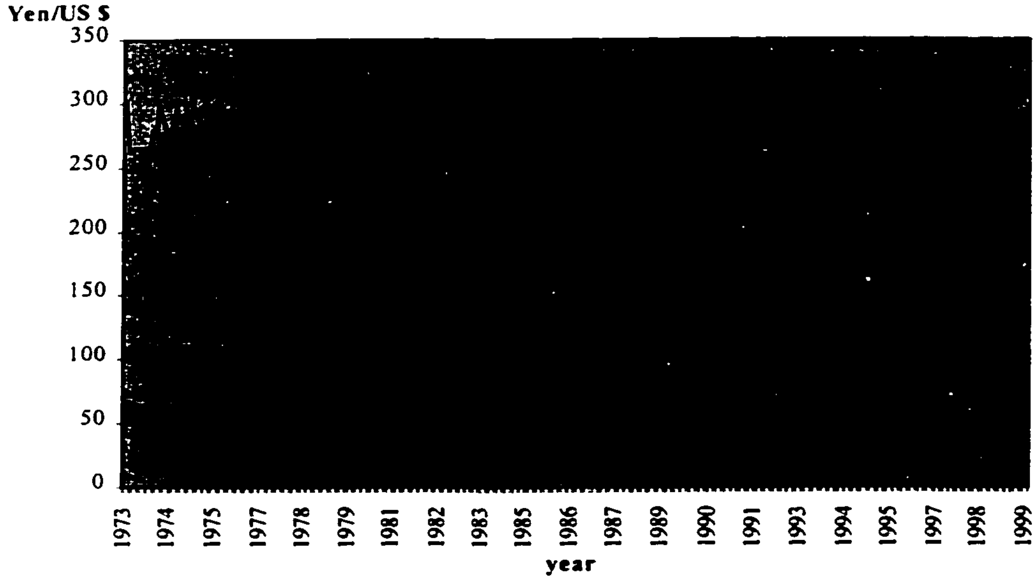
# CHAPTER 5
# DATA AND EMPIRICAL RESULTS OF TESTING PPP

## Data

The data for this study were obtained from International Financial Statistics (IFS) and are monthly running from January 1973 to October 1999. The exchange rates used are bilateral U.S. dollar rates of the Belgium franc, Canada dollar, Denmark krone, France franc, Germany mark, Italy lira, Japan yen, Netherlands guilder, Norway krone, Spain peseta, Sweden krona and U.K. pound. All these data are from line RF except the U.K., which is in line RH.[11] Figures 5.1 – 5.2 display the time paths of exchange rates and exchange rate returns of Japan and United Kingdom between January 1973 and October 1999.

For the price level, in previous research that has used both CPIs and WPIs similar results were found. In this study, the price level for all countries listed above is measured by the consumer price index in line 64 of IFS data, also although we try the wholesale price index in line 63 of IFS data for a few countries – Japan, United kingdom, and United States for comparison. All the price indices are transformed to make 1995 the base year. Figures 5.3 – 5.4 display the time paths and logarithmic price changes of the CPI of Japan and United Kingdom. Informally we can see some extreme outlier behavior from the logarithmic change figures for both exchange rates and price levels. That is, they may have fat-tail distributions.

[11] All currencies are originally expressed as the home currency per unit of foreign currency, except for the U.K. pound. The reciprocal of the published U.K. pound rate was utilised.



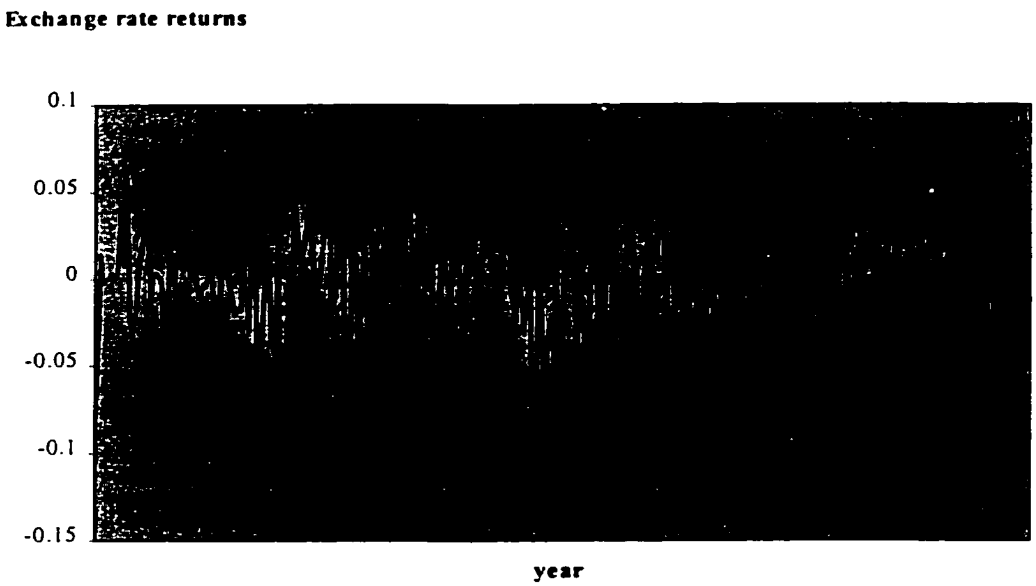
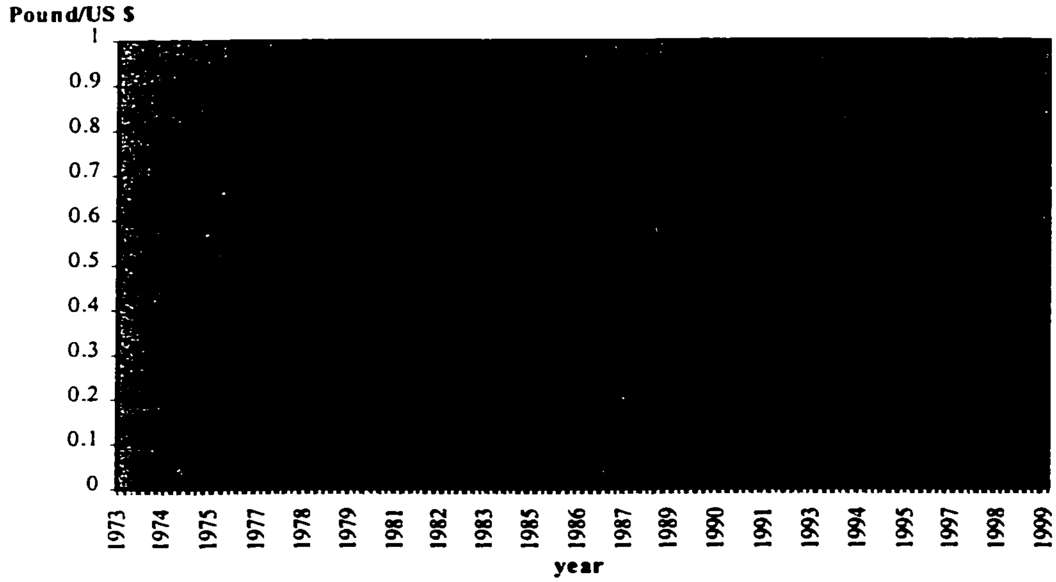




Figure 5.1. The exchange rate of the Japan yen expressed in U.S. dollar and the exchange rate returns, January 1973 – October 1999.



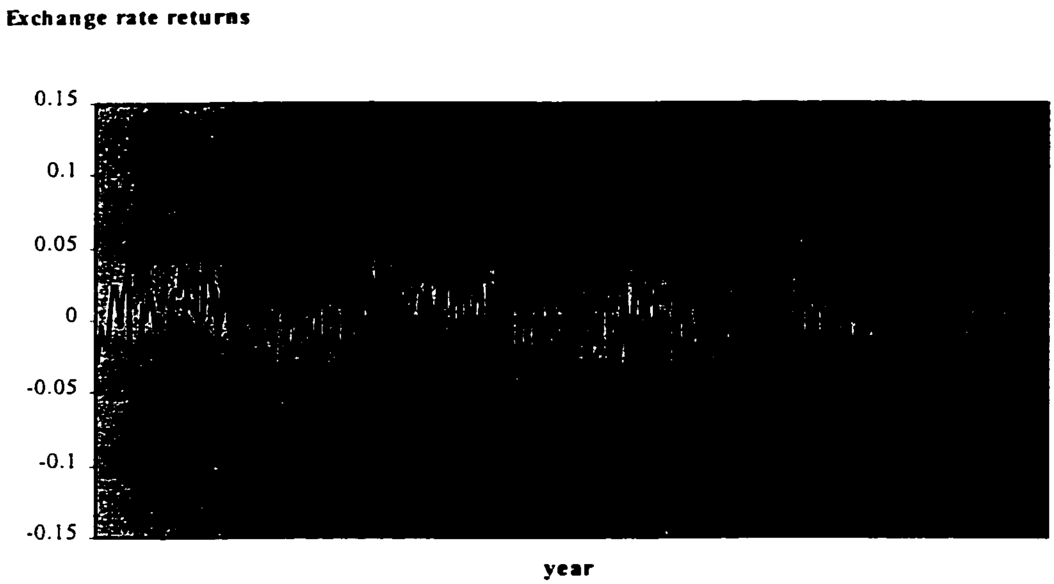
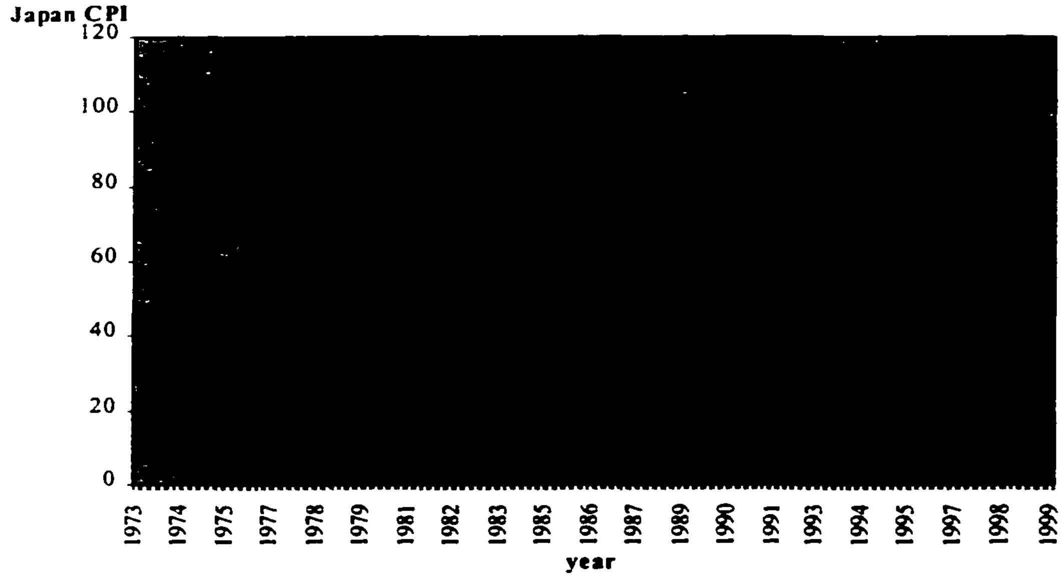




Figure 5.2. The exchange rate of the U.K. pound expressed in U.S. dollar and the exchange rate returns, January 1973 – October 1999.



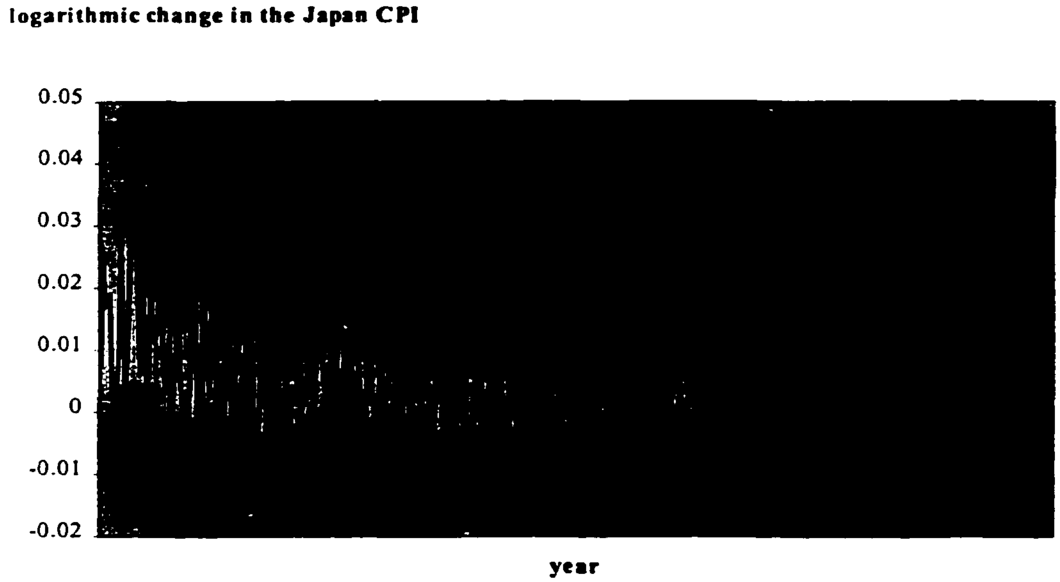
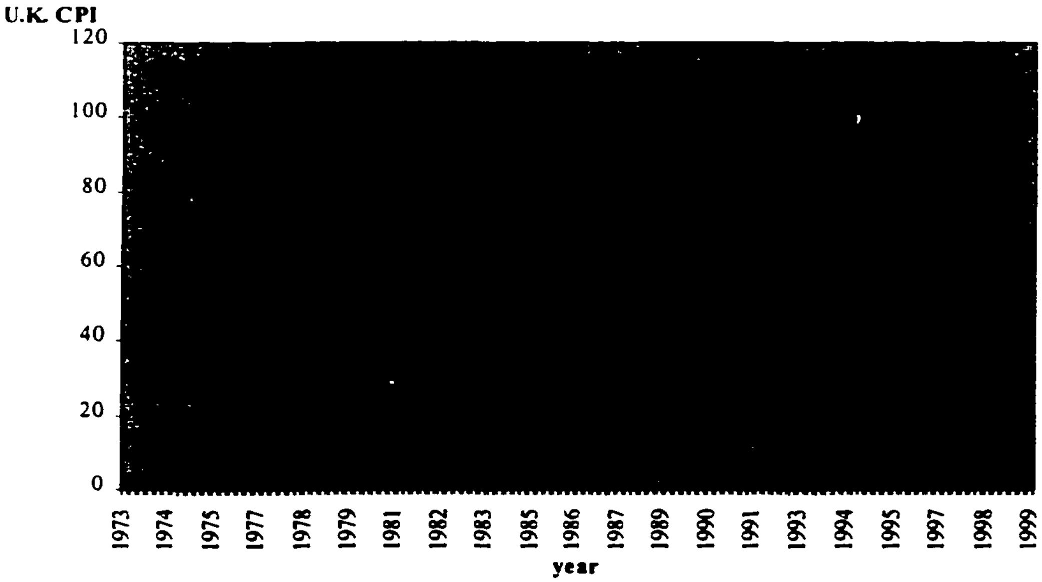




Figure 5.3. The CPI of Japan and the logarithmic change, between January 1973 and October 1999.



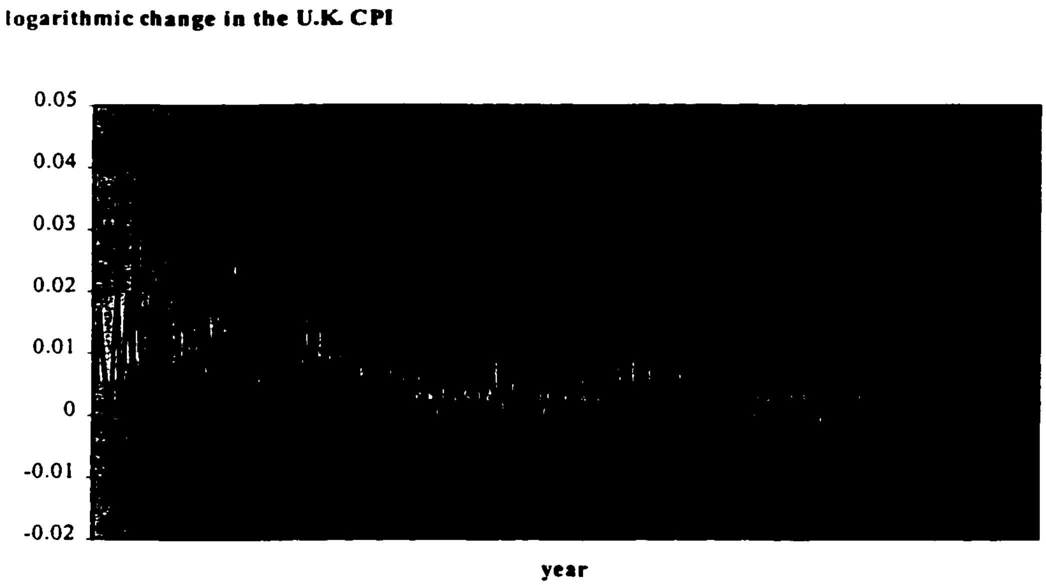




Figure 5.4. The CPI of U.K. and the logarithmic changes, between January 1973 and October 1999.

## Unit Root Tests

Under the assumption of stable errors, we implement the Phillips-Perron (1988) procedures with Caner's (1998) critical values for the unit root tests in exchange rates and price indices. The estimated Phillips-Perron statistics along with Caner's critical values are reported in Table 5.1 and 5.2 for the levels and first differences of the exchange rates and price indices, respectively. All of the exchange rate series appear to have a single unit root in that although we cannot reject the null of a unit root when these variables appear in levels, we can reject the null when the variables enter as first differences. Deterministic time trends do not appear to be important for these exchange rate variables, but they are important for the price indices. For the CPI series, two (Japan and Netherlands) appear stationary in levels. For the remaining series, they appear to have a single unit root since we cannot reject the null of a unit root in the levels but can reject the null in the first differences. For the WPI series, two (United Kingdom and United States) have similar results to their CPI series, but one (Japan) appears to have a single unit root, while its CPI series is stationary. Even though our exchange rate series and price series may be integrated of different orders, it is still possible for these variables to interact in such a way as to produce an I(0) series, i.e. a stationary series.

## Estimation of Stability Index

In conventional time series study, if a time series $\{X_t\}$~i.i.d.$(0,\sigma^2)$, then $\rho(j)=0$ if $|j|>0$. Thus for a size n time series, if we plot the sample autocorrelation function $\tilde{\rho}(k)$ as a

**Table 5.1. Phillips-Perron test statistic for a unit root test in exchange rates**

| | Level | | First difference |
|---|---|---|---|
| **Country** | **Trend** | **No Trend** | **No Trend** |
| Belgium | -1.75823 | -1.81976 | -12.63172* |
| Canada | -1.58447 | -0.96967 | -15.04193* |
| Denmark | -1.67657 | -1.66161 | -12.86491* |
| France | -1.61533 | -1.53693 | -13.26301* |
| Germany | -2.19173 | -2.27226 | -12.83553* |
| Italy | -1.76007 | -1.77043 | -11.68581* |
| Japan | -2.22643 | -1.00071 | -12.39299* |
| Netherlands | -2.03106 | -2.10669 | -12.77213* |
| Norway | -2.47705 | -1.50044 | -12.38141* |
| Spain | -1.40626 | -1.11012 | -12.36307* |
| Sweden | -1.91733 | -1.08657 | -11.41707* |
| United Kingdom | -2.13878 | -2.16861 | -11.86849* |

January 1973 – October 1999 of monthly nominal exchange rates (in U.S. dollar) from IMF.

* =rejecting the null hypothesis of a unit root at 5% significant level when $\alpha=1$ & 1.5

The 5% Caner's critical values for no trend and trend are, respectively, -3.40 and -3.84 when $\alpha=1.5$

The 5% Caner's critical values for no trend and trend are, respectively, -3.95 and -4.23 when $\alpha=1$

**Table 5.2. Phillips-Perron test statistic for a unit root test in price index**

| Price Index | Country | Level<br>Trend | First difference<br>No Trend |
|---|---|---|---|
| Consumer Price Index | Belgium | -2.54304 | -10.23826* |
| | Canada | -0.65740 | -12.05091* |
| | Denmark | -2.07753 | -13.70377* |
| | France | -0.62701 | -5.80118* |
| | Germany | -2.05852 | -11.74822* |
| | Italy | -0.53632 | -6.45621* |
| | Japan | -8.15776* | -12.72856* |
| | Netherlands | -5.24912* | -12.18042* |
| | Norway | 0.44271 | -15.06961* |
| | Spain | 0.12872 | -15.52796* |
| | Sweden | 1.75848 | -14.68597* |
| | United Kingdom | -2.59687 | -10.59010* |
| | United States | -1.60725 | -8.28111* |
| Wholesale Price Index | Japan | -3.49040 | -7.15236* |
| | United Kingdom | 0.07268 | -15.37883* |
| | United States | -1.66814 | -12.08778* |

January 1973 – October 1999 of monthly CPI and WPI from IMF.

* =rejecting the null hypothesis of a unit root at 5% significant level when $\alpha$=1 & 1.5

the 5% Caner's critical values for no trend and trend are, respectively, -3.40 and -3.84 when $\alpha$=1.5

the 5% Caner's critical values for no trend and trend are, respectively, -3.95 and -4.23 when $\alpha$=1 function of k, approximately 0.95 of the sample autocorrelations should lie between the bounds $\pm 1.96n^{-1/2}$, i.e. Gaussian based limits if the $X_t^{'}$s are normally distributed or n is sufficiently large. This can be used as a check that the observations truly are from an i.i.d. process.

Based on the Adler, Feldman, and Gallagher (1998) study, we use Box-Jenkins methods with Cauchy based limits instead of Gaussian based limits for model identification and diagnostic checking of i.i.d. residuals. For example, in Figures 5.5 and 5.6 we plot the ACF and PACF of U.K. pound/U.S. dollar exchange rate return series along with Gaussian based limits and Cauchy based limits respectively. Both figures show that these exchange rate returns are not i.i.d., since $\tilde{\rho}(0)$ is not the only one outside the confidence limits. Therefore, We fit this series to an MA(1). The ACF and PACF are plotted in Figures 5.7 and 5.8 along with Gaussian based limits and Cauchy based limits for the residuals of the MA(1) model. Since both figures have all ACF and PACF except first lag within those confidence limits, the assumption of i.i.d. innovations is not rejected. In fact all of the exchange rate return series were identified as MA(1) processes. This identification procedure was also applied to the inflation series. It turned out that most of these series required a twelve-order AR term to remove the serial correlation in the residuals. This may reflect a seasonal effect in the monthly inflation rates. The models selected for the inflation rates are provided in Table 5.3.

After obtaining a model with i.i.d. errors, we obtain the maximum likelihood estimates of the stability parameters for the residual series obtained from the fitted model by running the program STABLE in Nolan (1997). The summary of results is contained in Table 5.3.

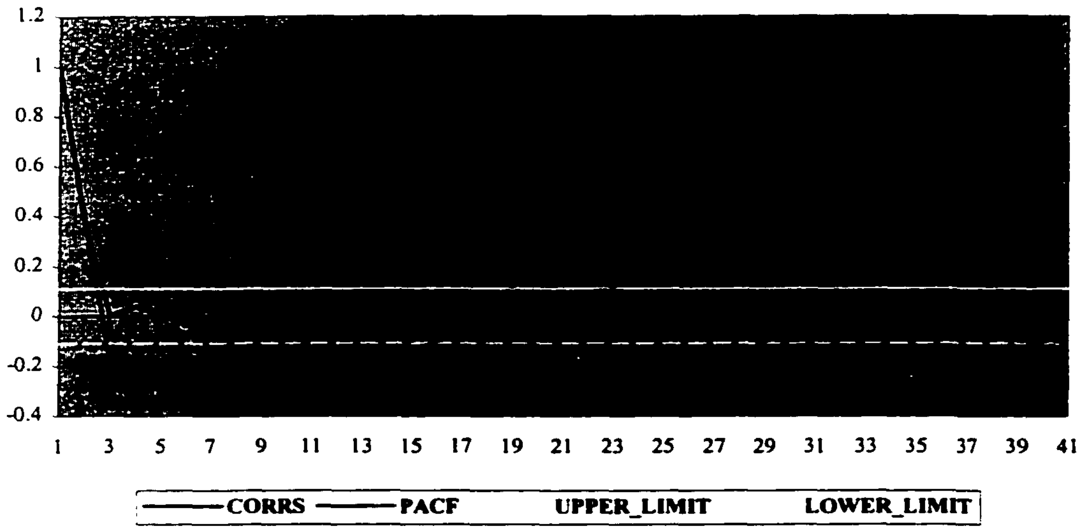



**Figure 5.5. ACF and PACF of U.K. pound/U.S. dollar exchange rate returns with Gaussian limits.**

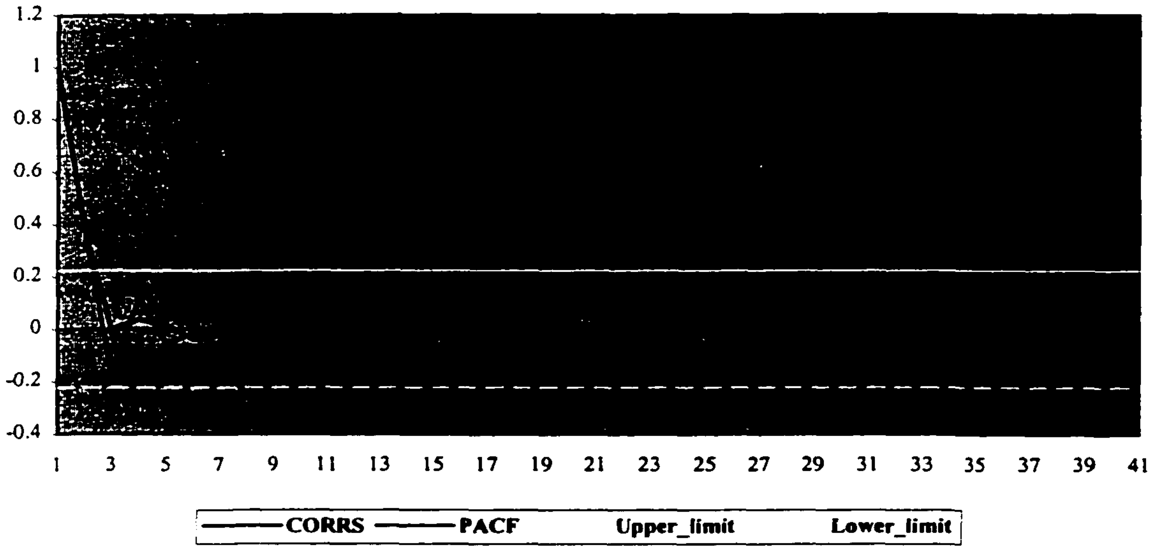



**Figure 5.6. ACF and PACF of U.K. pound/U.S. dollar exchange rate returns with Cauchy limits.**

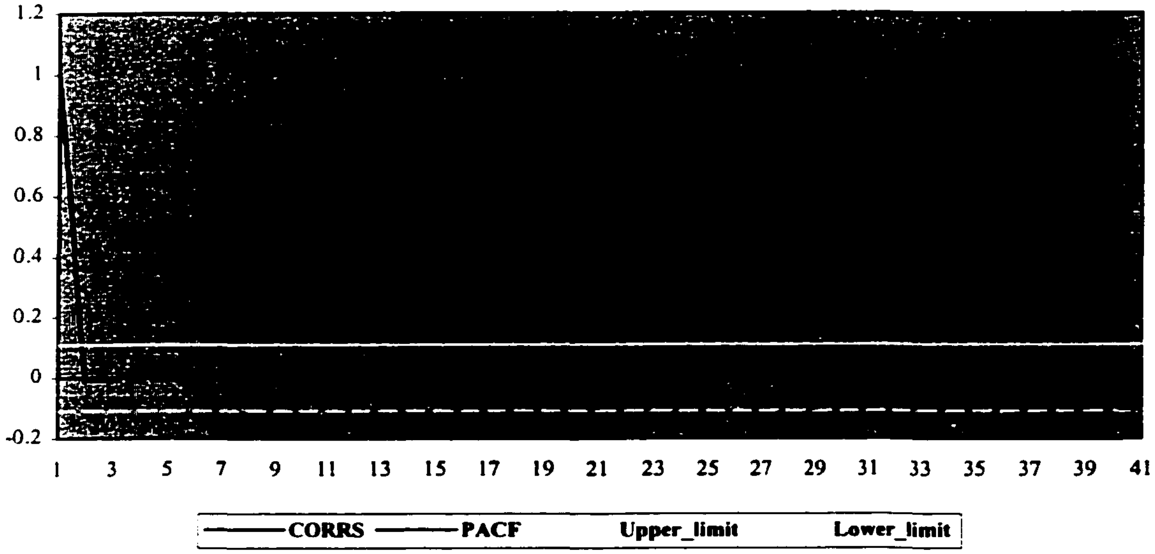



**Figure 5.7. ACF and PACF of the residuals in the MA(1) model with Gaussian limits.**

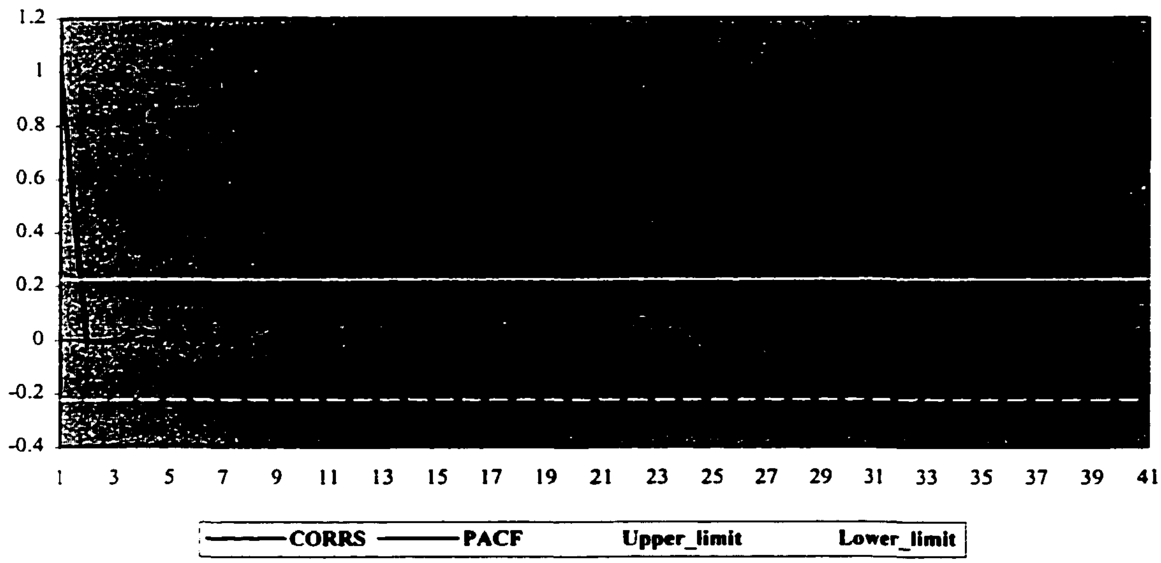



**Figure 5.8. ACF and PACF of the residuals in the MA(1) model with Cauchy limits.**

**Table 5.3 Stability index (α) estimates and confidence intervals**

| Variable X | Country | ln(X(t)/X(t-1)) fitted model | α | 95% C.I. | 90% C.I. |
|---|---|---|---|---|---|
| Exchange rate | Belgium | MA(1) | 1.8260 | ±0.1446 | ±0.1214 |
| | Canada | MA(1) | 1.9666 | ±0.0754 | ±0.0633 |
| | Denmark | MA(1) | 1.8723 | ±0.1282 | ±0.1076 |
| | France | MA(1) | 1.9193 | ±0.1033 | ±0.0867 |
| | Germany | MA(1) | 1.8055 | ±0.1482 | ±0.1244 |
| | Italy | MA(1) | 1.6488 | ±0.1694 | ±0.1422 |
| | Japan | MA(1) | 1.7108 | ±0.1563 | ±0.1312 |
| | Netherlands | MA(1) | 1.9035 | ±0.1221 | ±0.1025 |
| | Norway | MA(1) | 1.6636 | ±0.1688 | ±0.1417 |
| | Spain | MA(1) | 1.5412 | ±0.1738 | ±0.1459 |
| | Sweden | MA(1) | 1.7789 | ±0.1546 | ±0.1298 |
| | United Kingdom | MA(1) | 1.6462 | ±0.1701 | ±0.1428 |
| CPI | Belgium | AR(‖1,12‖) | 1.8216 | ±0.1494 | ±0.1254 |
| | Canada | AR(‖1,12‖) | 1.5309 | ±0.1768 | ±0.1484 |
| | Denmark | AR(‖12‖) | 1.5692 | ±0.1741 | ±0.1461 |
| | France | AR(‖1,12‖) | 1.5921 | ±0.1745 | ±0.1465 |
| | Germany | AR(‖1,12‖) | 1.4843 | ±0.1751 | ±0.1470 |
| | Italy | AR(‖1,12‖) | 1.3703 | ±0.1729 | ±0.1451 |
| | Japan | AR(‖12‖) | 1.6163 | ±0.1743 | ±0.1463 |
| | Netherlands | AR(‖12‖) | 1.6388 | ±0.1714 | ±0.1439 |
| | Norway | AR(‖1,12‖) | 1.4916 | ±0.1758 | ±0.1475 |
| | Spain | AR(1) | 1.4046 | ±0.1601 | ±0.1344 |
| | Sweden | AR(‖1,12‖) | 1.3041 | ±0.1681 | ±0.1411 |
| | United Kingdom | AR(‖1,12‖) | 1.4508 | ±0.1735 | ±0.1456 |
| | United States | AR(2) | 1.6509 | ±0.1681 | ±0.1411 |
| WPI | Japan | AR(1) | 1.5917 | ±0.1668 | ±0.1400 |
| | United Kingdom | AR(12) | 1.5819 | ±0.1751 | ±0.1470 |
| | United States | AR(12) | 1.8137 | ±0.1605 | ±0.1347 |

CPI: Consumer Price Index

WPI: Wholesale Price Index

C.I.: confidence interval

January 1973 – October 1999 of monthly CPI, WPI (1995=100) & nominal exchange rates from IMF.

We find that the point estimates of $\alpha$ for logarithmic price changes are less than 2 for both consumer price index and wholesale price index, also the 95% and 90% confidence intervals always exclude 2. That is, all the logarithmic price changes appear to have the non-Gaussian behavior. For exchange rate returns, 95% and 90% confidence intervals include $\alpha$=2 in only 3 out of 12 cases. Thus most of the exchange rate returns also show evidence of non-Gaussian behavior. Therefore, the assumption of stable errors, i.e. infinite variance errors, may be more prudent for our data than the assumption of finite variance errors.

**Residual-based Cointegration Tests**

The residual-based cointegration tests of PPP were implemented by estimating equation (3.1). The test statistics are contained in Table 5.4. We cannot reject the null hypothesis of a unit root in the estimated residual series, i.e. no cointegration in those variables – $s$, $p$ and $p^*$, in any of the cases. This does not depend on whether we use conventional Engle-Yoo's critical values or Caner's critical values. That is, no matter whether we assume finite variance errors or infinite variance errors, these results fail to provide evidence of a long-run PPP relationship. The results are consistent to the previous studies about testing the PPP long-run relationship by Engle-Granger cointegration method. Moreover, it also consistent with Caner's (1998) general result – if the researchers falsely assume that error terms are square integrable the residual-based test statistics will slightly over-reject the null of no cointegration, because the Engel-Yoo's critical value is less than the Caner's critical value. However, because of the drawbacks of the residual-based cointegration method mentioned in chapter 3, even under the assumption of stable errors we still should look further for evidence of supporting long-run PPP relationship.

## Multivariate Cointegration Tests

Two approaches were used to select the lag length k. In the first approach, k is determined by minimizing the multivariate Akaike information criterion (AIC)[12], which is the adjusted test criterion for a VAR model. For diagnostic checking, we use the Box-Jenkins methodology with Gaussian based limits (for applications assuming normal errors) and Cauchy limits (for applications assuming stable errors). All of our multivariate cointegration tests are implemented using a constant and eleven seasonal dummies in the underlying VAR model for capturing seasonality.

The second approach was to follow MacDonald (1993), who implemented Johansen's multivariate cointegration tests using a twelfth-order lag in the underlying VAR, which also contained a constant and eleven seasonal dummies. The chosen lag for this model was assumed to be sufficient to ensure residual whiteness and account for any seasonality not captured by the seasonal dummies.

In tables 5.5 and 5.6 we report our estimates of the maximum eigenvalue and trace statistics (equations 3.7 and 3.6), respectively, along with the ranks, when we use a twelfth-order lag and twelve seasonal dummies in the underlying VAR. In those two tables, for tests

[12] The multivariate generation of AIC $= T\log|\Sigma| + 2N$, where T is the number of useful observations, $|\Sigma|$ =determinant of the variance/covariance matrix of the residuals and N=total number of parameters estimated in all equations.

**Table 5.4 Residual-based cointegration tests in PPP**

| | Country | No Trend | Trend |
|---|---|---|---|
| CPI | Belgium | -2.08078 | -1.99974 |
| | Canada | -1.79623 | -1.77925 |
| | Denmark | -2.27722 | -2.21609 |
| | France | -2.18764 | -2.15783 |
| | Germany | -2.09811 | -2.04118 |
| | Italy | -1.99435 | -1.98136 |
| | Japan | -2.28473 | -2.25890 |
| | Netherlands | -2.53681 | -2.46826 |
| | Norway | -2.72961 | -2.68191 |
| | Spain | -1.93912 | -1.90926 |
| | Sweden | -2.32562 | -2.30424 |
| | United Kingdom | -2.36558 | -2.36786 |
| WPI | Japan | -3.30982 | -3.35439 |
| | United Kingdom | -2.30427 | -2.34206 |

* =rejecting the null hypothesis of no cointegration ( i.e. a unit root) at 5% significant level
the 5% Engle-Yoo's critical value is -3.78
the 5% Caner's critical values for no trend and trend are, respectively, -4.23 and -4.52 when $\alpha=1.5$
the 5% Caner's critical values for no trend and trend are, respectively, -4.93 and -5.28 when $\alpha=1$

of size 5% and 2.5%, Osterwald-Lenum's (1992)[13] critical values and Caner's critical values for stability index $\alpha$=1.5 are reported for comparison. According to the ranks, i.e., the number of cointegrating vectors, shown in those tables, the number decided by Caner's critical values is equal to or smaller than the one decided by Osterwalk-Lenum's critical values, for a test of size 5% or 2.5%. This finding is consistent with Caner (1998), which concluded that when the stability index $\alpha$<2, using Johansen's (or Osterwalk-Lenum's) critical values biased the tests in favor of more cointegrating vectors. Moreover, on the basis of maximum eigenvalue statistics, zero cointegrating vectors, that is PPP not holding, happens to eight of twelve countries according to Caner's critical values while it only happens to five of the twelve countries according to Osterwalk-Lenum's critical values. These eight countries include Canada, Denmark, France, Germany, Italy, Norway, Spain and Sweden while Denmark, France and Spain are excluded in the other group. They do not have a long-run PPP relationship with the United States in this case. On the basis of trace statistics, no cointegrating vector appears for four (Canada, Germany, Italy and Sweden) of twelve countries according to Caner's critical values, while it only appears to one (Sweden) of them according to Osterwalk-Lenum's critical values. So, when Caner's critical values are used the results are less supportive of weak-form PPP.

In tables 5.7 and 5.8 we report our estimates of the maximum eigenvalue and trace statistics (equations 3.7 and 3.6), respectively, along with the ranks for test of size 5% and 2.5%, when the innovations are assumed to be Gaussian and the VAR lag length are selected

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[13] Obsterwald-Lenum (1992) provides the extended versions of the four tables presented in Johansen (1988) and Johansen and Juselius (1990).

using the multivariate AIC. On the basis of maximum eigenvalue statistics, eight out of twelve combinations show evidence of at least one cointegrating vector. Those combinations include Belgium, Denmark, France, Italy, Japan, Netherlands, Spain, and United Kingdom against United States. On the basis of trace statistics, all but Germany have at least one cointegrating vector. These results appear to provide support for weak-form PPP, consistent with the results in previous research.

In tables 5.9 and 5.10, we report the maximum eigenvalue and trace statistics with the ranks decided by 5% and 2.5% critical values for stability index $\alpha=1.5$ and $\alpha=1.1$ from Caner (1998) with the lag length chosen by the AIC and diagnostics performed assuming Cauchy process. On the basis of both maximum eigenvalue and trace statistics, all cases except for Canada have at least one cointegrating vector using the stability index $\alpha=1.5$. That is, under the assumption of stability index $\alpha=1.5$, all countries except Canada have a weak-form PPP relationship with the United States. But, under the assumption of stability index $\alpha=1.1$, the number of cases with zero cointegrating vectors increases to four (Canada, France, Netherlands and Norway) out of twelve on the basis of maximum eigenvalue statistics, while not changeing on the basis of trace statistics. Thus, under the assumption of stability index $\alpha=1.1$, we find the evidence less supportive of the weak-form PPP relationship with United States. This finding shows that the results are sensitive to the assumption of the stability index. The multivariate cointegration method in Caner (1998) requires that the error terms in equation (4.15) satisfies the assumption of being i.i.d, symmetrically distributed and in the normal domain of attraction of a tri-variate symmetric stable law with the same stability index $\alpha$, for all components of the vector, $0<\alpha<2$. However, due to the limitation of testing the equality of the stability index $\alpha$, we can only assume the equality of stability index and choose the closest value of $\alpha$. As shown in table 5.3, the estimated stability index for CPIs are ranged from 1.30 to 1.82, for WPIs from 1.58 to 1.81, and for nominal exchange rates from 1.64 to 1.96. Thus the assumption of stability index $\alpha=1.5$ may be more appropriate than $\alpha=1.1$.

Comparing tables 5.7 and 5.9, on the basis of maximum eigenvalue statistics, the numbers of combinations having at least one unique cointegrating vector are eight and eleven (out of twelve) respectively. That is, the results under the assumption of stable errors show more support for weak-form PPP than the results under the assumption of normal errors. Therefore, under the assumption of stable errors, we find stronger evidence of long-run PPP relationship with United States.

Above cointegration analysis for PPP relationship is based on CPI. In order to see the effect of the conintegration analysis by the different price index, we try wholesale price index (WPI) for a few countries – Japan, United Kingdom and United States. Tables 5.11-5.16 provide the test statistics and the number of rank as shown in tables 5.5-5.10 when the price level in the PPP relationship is WPI instead of CPI. According to tables 5.5, 5.6, 5.11 and 5.12, the number of cointegrating vector in the United Kingdom-United States case remains the same when WPI is used instead of CPI, while the one in the Japan-United States case becomes smaller. Moreover, in the Japan-United States case, the evidence of supporting weak-form PPP relationship becomes invalid. In tables 5.7, 5.8, 5.13 and 5.14 the results still remain the same in both cases, even though in the Japan-United States case the number of rank based on WPI is different from the one based on CPI, but not equal to zero. That is, based the conventional Johansen cointegration tests the different price indexes do not have different effect in our study. However, based on the cointegration tests with stable error assumption, the effect is shown. From tables 5.9, 5.10, 5.15 and 5.16 the evidence of supporting the weak-form PPP relationship become rejecting in the Japan-United States case, even though the results are the same in the United Kingdom-United States case.

The theory of strong-form PPP imposed symmetry and proportionality conditions are on the long-run coefficients, which imply that ($s - p + p^*$) is stationary rather than a unit root process. Table 5.17 contains the statistical results from testing the null hypothesis that ($s - p + p^*$) is a unit root process against the stationary alternative. In no case can we find supportive evidence for strong-form PPP. Finding non-stationarity in the univariate model indicates a violation of the symmetry or proportionaltiy restriction. The violation of strong-form PPP may reflect the existence of transportation costs, trade impediments, product differentiation and the presence of non-traded goods.

Table 5.5. **Maximum eigenvalue test statistics for the hypothesis about the number of cointegrating vectors in PPP**

| Country | $H_0$ | Maximum eigenvalue | Gaussian Rank(5%) | Gaussian Rank(2.5%) | Non-Gaussian Rank(5%) | Non-Gaussian Rank(2.5%) |
|---|---|---|---|---|---|---|
| Belgium | r=0 | 46.25 | 2 | 2 | 2 | 1 |
| | r=1 | 19.15 | | | | |
| | r=2 | 5.32 | | | | |
| Canada | r=0 | 16.90 | 0 | 0 | 0 | 0 |
| | r=1 | 12.59 | | | | |
| | r=2 | 3.07 | | | | |
| Denmark | r=0 | 24.05 | 1 | 1 | 0 | 0 |
| | r=1 | 11.67 | | | | |
| | r=2 | 2.62 | | | | |
| France | r=0 | 23.24 | 1 | 1 | 0 | 0 |
| | r=1 | 10.36 | | | | |
| | r=2 | 3.98 | | | | |
| Germany | r=0 | 18.48 | 0 | 0 | 0 | 0 |
| | r=1 | 10.00 | | | | |
| | r=2 | 3.39 | | | | |
| Italy | r=0 | 20.82 | 0 | 0 | 0 | 0 |
| | r=1 | 8.03 | | | | |
| | r=2 | 5.10 | | | | |
| Japan | r=0 | 41.28 | 1 | 1 | 1 | 1 |
| | r=1 | 13.36 | | | | |
| | r=2 | 9.00 | | | | |
| Netherlands | r=0 | 26.37 | 3 | 2 | 1 | 0 |
| | r=1 | 17.24 | | | | |
| | r=2 | 8.37 | | | | |
| Norway | r=0 | 15.88 | 0 | 0 | 0 | 0 |
| | r=1 | 11.01 | | | | |
| | r=2 | 8.70 | | | | |
| Spain | r=0 | 24.42 | 1 | 1 | 0 | 0 |
| | r=1 | 11.70 | | | | |
| | r=2 | 3.64 | | | | |
| Sweden | r=0 | 17.28 | 0 | 0 | 0 | 0 |
| | r=1 | 10.86 | | | | |
| | r=2 | 2.72 | | | | |
| United Kingdom | r=0 | 29.47 | 3 | 3 | 3 | 0 |
| | r=1 | 17.88 | | | | |
| | r=2 | 10.52 | | | | |

The underlying vector autoregressions included 12 seasonal dummies and twelfth-order lag.

r: ranks (i.e. number of cointegrating vectors)

In Gaussian case, critical values for a test of size 5% are 21.07, 14.90 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are 22.89, 17.07 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).
In non-Gaussian case, critical values for a test of size 5% are 24.50, 16.82 and 8.74 for r=0, r=1 and r=2 respectively, and ones for a test of size 2.5% are 29.77, 20.66 and 11.34 for r=0, r=1 and r=2 respectively, from Caner (1998).

Table 5.6. **Trace test statistics for the hypothesis about the number of cointegrating vectors in PPP**

| | | | Gaussian | | Non-Gaussian | |
|---|---|---|---|---|---|---|
| Country | $H_0$ | Trace | Rank(5%) | Rank(2.5%) | Rank(5%) | Rank(2.5%) |
| Belgium | r=0 | 70.72 | 2 | 2 | 2 | 2 |
| | r<=1 | 24.46 | | | | |
| | r<=2 | 5.32 | | | | |
| Canada | r=0 | 32.55 | 1 | 0 | 0 | 0 |
| | r<=1 | 15.65 | | | | |
| | r<=2 | 3.07 | | | | |
| Denmark | r=0 | 38.25 | 1 | 1 | 1 | 0 |
| | r<=1 | 14.30 | | | | |
| | r<=2 | 2.62 | | | | |
| France | r=0 | 37.58 | 1 | 1 | 1 | 0 |
| | r<=1 | 14.34 | | | | |
| | r<=2 | 3.98 | | | | |
| Germany | r=0 | 31.88 | 1 | 0 | 0 | 0 |
| | r<=1 | 13.39 | | | | |
| | r<=2 | 3.39 | | | | |
| Italy | r=0 | 33.95 | 1 | 0 | 0 | 0 |
| | r<=1 | 13.13 | | | | |
| | r<=2 | 5.10 | | | | |
| Japan | r=0 | 63.64 | 3 | 3 | 3 | 1 |
| | r<=1 | 22.36 | | | | |
| | r<=2 | 9.00 | | | | |
| Netherlands | r=0 | 51.97 | 3 | 2 | 2 | 1 |
| | r<=1 | 25.61 | | | | |
| | r<=2 | 8.37 | | | | |
| Norway | r=0 | 35.59 | 3 | 1 | 1 | 0 |
| | r<=1 | 19.71 | | | | |
| | r<=2 | 8.70 | | | | |
| Spain | r=0 | 39.76 | 1 | 1 | 1 | 0 |
| | r<=1 | 15.34 | | | | |
| | r<=2 | 3.64 | | | | |
| Sweden | r=0 | 30.87 | 0 | 0 | 0 | 0 |
| | r<=1 | 13.58 | | | | |
| | r<=2 | 2.72 | | | | |
| United Kingdom | r=0 | 57.88 | 3 | 3 | 3 | 2 |
| | r<=1 | 28.41 | | | | |
| | r<=2 | 10.52 | | | | |

The underlying vector autoregressions included 12 seasonal dummies and twelfth-order lag.

r: ranks (i.e. number of cointegrating vectors)

In Gaussian case, critical values for a test of size 5% are 31.52, 17.95 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are, 34.48, 20.08 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).
In non-Gaussian case, critical values for a test of size 5% are 34.99, 19.90 and 8.74 for r=0, r=1 and r=2 respectively, and ones for a test of size 2.5% are 40.71, 24.22 and 11.34 for r=0, r=1 and r=2 respectively, from Caner (1998).

**Table 5.7. Maximum eigenvalue test statistics for Johansen's multivariate cointegration tests in PPP**

| Country | Lag | $H_0$ | Maximum eigenvalue | Rank(5%) | Rank(2.5%) |
|---|---|---|---|---|---|
| Belgium | 10 | r=0 | 48.69 | 2 | 2 |
| | | r=1 | 18.30 | | |
| | | r=2 | 6.98 | | |
| Canada | 12 | r=0 | 16.90 | 0 | 0 |
| | | r=1 | 12.59 | | |
| | | r=2 | 3.07 | | |
| Denmark | 8 | r=0 | 27.73 | 2 | 1 |
| | | r=1 | 17.02 | | |
| | | r=2 | 5.04 | | |
| France | 10 | r=0 | 21.38 | 1 | 0 |
| | | r=1 | 12.75 | | |
| | | r=2 | 4.26 | | |
| Germany | 10 | r=0 | 20.76 | 0 | 0 |
| | | r=1 | 5.94 | | |
| | | r=2 | 2.94 | | |
| Italy | 10 | r=0 | 22.02 | 1 | 0 |
| | | r=1 | 10.40 | | |
| | | r=2 | 4.83 | | |
| Japan | 10 | r=0 | 64.36 | 1 | 1 |
| | | r=1 | 14.83 | | |
| | | r=2 | 7.87 | | |
| Netherlands | 10 | r=0 | 28.85 | 2 | 1 |
| | | r=1 | 16.97 | | |
| | | r=2 | 5.68 | | |
| Norway | 10 | r=0 | 19.01 | 0 | 0 |
| | | r=1 | 12.11 | | |
| | | r=2 | 9.76 | | |
| Spain | 10 | r=0 | 28.16 | 1 | 1 |
| | | r=1 | 11.66 | | |
| | | r=2 | 4.05 | | |
| Sweden | 10 | r=0 | 17.74 | 0 | 0 |
| | | r=1 | 11.03 | | |
| | | r=2 | 3.62 | | |
| United Kingdom | 11 | r=0 | 32.19 | 1 | 1 |
| | | r=1 | 13.94 | | |
| | | r=2 | 8.50 | | |

The underlying vector autoregressions included 12 seasonal dummies and different lags.

Lag: number of lags chosen based on AIC & Gaussian limits of residuals.

The critical values for a test of size 5% are 21.07, 14.90 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are 22.89, 17.07 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).

r: rank (i.e. number of cointegrating vectors)

**Table 5.8. Trace test statistics for Johansen's multivariate cointegration tests in PPP**

| Country | Lag | $H_0$ | Trace | Rank(5%) | Rank(2.5%) |
|---|---|---|---|---|---|
| Belgium | 10 | r=0 | 73.98 | 2 | 2 |
| | | r<=1 | 25.29 | | |
| | | r<=2 | 6.98 | | |
| Canada | 12 | r=0 | 32.55 | 1 | 0 |
| | | r<=1 | 15.65 | | |
| | | r<=2 | 3.07 | | |
| Denmark | 8 | r=0 | 49.79 | 2 | 2 |
| | | r<=1 | 22.06 | | |
| | | r<=2 | 5.04 | | |
| France | 10 | r=0 | 38.39 | 1 | 1 |
| | | r<=1 | 17.01 | | |
| | | r<=2 | 4.26 | | |
| Germany | 10 | r=0 | 29.64 | 0 | 0 |
| | | r<=1 | 8.88 | | |
| | | r<=2 | 2.94 | | |
| Italy | 10 | r=0 | 37.25 | 1 | 1 |
| | | r<=1 | 15.23 | | |
| | | r<=2 | 4.83 | | |
| Japan | 10 | r=0 | 87.07 | 2 | 2 |
| | | r<=1 | 22.71 | | |
| | | r<=2 | 7.87 | | |
| Netherlands | 10 | r=0 | 51.51 | 2 | 2 |
| | | r<=1 | 22.65 | | |
| | | r<=2 | 5.68 | | |
| Norway | 10 | r=0 | 40.88 | 3 | 3 |
| | | r<=1 | 21.87 | | |
| | | r<=2 | 9.76 | | |
| Spain | 10 | r=0 | 43.86 | 1 | 1 |
| | | r<=1 | 15.70 | | |
| | | r<=2 | 4.05 | | |
| Sweden | 10 | r=0 | 32.39 | 1 | 0 |
| | | r<=1 | 14.65 | | |
| | | r<=2 | 3.62 | | |
| United Kingdom | 11 | r=0 | 54.63 | 3 | 2 |
| | | r<=1 | 22.44 | | |
| | | r<=2 | 8.50 | | |

The underlying vector autoregressions included 12 seasonal dummies and different lags.

Lag: number of lags chosen based on AIC & Gaussian limits of residuals.

The critical values for a test of size 5% are 31.52, 17.95 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are, 34.48, 20.08 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).

r: rank (i.e. number of cointegrating vectors)

**Table 5.9. Maximum eigenvalue test statistics for Caner's multivariate cointegration tests in PPP**

| Country | Lag | $H_0$ | Maximum Eigenvalue | α=1.5 Rank(5%) | α=1.5 Rank(2.5%) | α=1.1 Rank(5%) | α=1.1 Rank(2.5%) |
|---|---|---|---|---|---|---|---|
| Belgium | 5 | r=0 | 52.13 | 3 | 1 | 1 | 1 |
| | | r=1 | 19.71 | | | | |
| | | r=2 | 10.93 | | | | |
| Canada | 11 | r=0 | 17.92 | 0 | 0 | 0 | 0 |
| | | r=1 | 11.47 | | | | |
| | | r=2 | 3.15 | | | | |
| Denmark | 6 | r=0 | 36.76 | 2 | 1 | 1 | 0 |
| | | r=1 | 19.08 | | | | |
| | | r=2 | 3.88 | | | | |
| France | 8 | r=0 | 31.11 | 1 | 1 | 0 | 0 |
| | | r=1 | 13.72 | | | | |
| | | r=2 | 4.57 | | | | |
| Germany | 3 | r=0 | 65.72 | 1 | 1 | 1 | 1 |
| | | r=1 | 4.19 | | | | |
| | | r=2 | 2.11 | | | | |
| Italy | 3 | r=0 | 41.54 | 3 | 1 | 1 | 0 |
| | | r=1 | 17.59 | | | | |
| | | r=2 | 8.90 | | | | |
| Japan | 3 | r=0 | 72.64 | 2 | 2 | 2 | 2 |
| | | r=1 | 50.33 | | | | |
| | | r=2 | 5.03 | | | | |
| Netherlands | 10 | r=0 | 28.85 | 2 | 0 | 0 | 0 |
| | | r=1 | 16.97 | | | | |
| | | r=2 | 5.68 | | | | |
| Norway | 7 | r=0 | 27.28 | 1 | 0 | 0 | 0 |
| | | r=1 | 12.45 | | | | |
| | | r=2 | 6.60 | | | | |
| Spain | 2 | r=0 | 112.19 | 1 | 1 | 1 | 1 |
| | | r=1 | 12.47 | | | | |
| | | r=2 | 6.74 | | | | |
| Sweden | 3 | r=0 | 68.34 | 1 | 1 | 1 | 1 |
| | | r=1 | 13.35 | | | | |
| | | r=2 | 2.87 | | | | |
| United Kingdom | 4 | r=0 | 55.02 | 1 | 1 | 1 | 1 |
| | | r=1 | 12.83 | | | | |
| | | r=2 | 6.22 | | | | |

The underlying vector autoregressions included 12 seasonal dummies and different lags.

Lag: number of lags chosen based on AIC & Cauchy limits of residuals.

For α=1.5, the Caner' critical values for a test of size 5% are 24.50, 16.82 and 8.74 for r=0, r=1 and r=2 respectively

For α=1.5, the Caner' critical values for a test of size 2.5% are 29.77, 20.66 and 11.34 for r=0, r=1 and r=2 respectively

For α=1.1, the Caner' critical values for a test of size 5% are 32.05, 20.27 and 9.50 for r=0, r=1 and r=2 respectively

For α=1.1, the Caner' critical values for a test of size 2.5% are 42.96, 27.72 and 13.36 for r=0, r=1 and r=2 respectively

**Table 5.10. Trace test statistics for Caner's multivariate cointegration tests in PPP**

| | | | | α=1.5 | | α=1.1 | |
|---|---|---|---|---|---|---|---|
| **Country** | **Lag** | **$H_0$** | **Trace** | **Rank(5%)** | **Rank(2.5%)** | **Rank(5%)** | **Rank(2.5%)** |
| Belgium | 5 | r=0 | 82.76 | 3 | 2 | 3 | 1 |
| | | r<=1 | 30.64 | | | | |
| | | r<=2 | 10.93 | | | | |
| Canada | 11 | r=0 | 32.54 | 0 | 0 | 0 | 0 |
| | | r<=1 | 14.63 | | | | |
| | | r<=2 | 3.15 | | | | |
| Denmark | 6 | r=0 | 59.72 | 2 | 1 | 1 | 1 |
| | | r<=1 | 22.96 | | | | |
| | | r<=2 | 3.88 | | | | |
| France | 8 | r=0 | 49.40 | 1 | 1 | 1 | 0 |
| | | r<=1 | 18.29 | | | | |
| | | r<=2 | 4.57 | | | | |
| Germany | 3 | r=0 | 72.01 | 1 | 1 | 1 | 1 |
| | | r<=1 | 6.29 | | | | |
| | | r<=2 | 2.11 | | | | |
| Italy | 3 | r=0 | 68.02 | 3 | 2 | 2 | 1 |
| | | r<=1 | 26.48 | | | | |
| | | r<=2 | 8.90 | | | | |
| Japan | 3 | r=0 | 128.00 | 2 | 2 | 2 | 2 |
| | | r<=1 | 55.35 | | | | |
| | | r<=2 | 5.03 | | | | |
| Netherlands | 10 | r=0 | 51.51 | 2 | 1 | 1 | 0 |
| | | r<=1 | 22.65 | | | | |
| | | r<=2 | 5.68 | | | | |
| Norway | 7 | r=0 | 46.32 | 1 | 1 | 1 | 0 |
| | | r<=1 | 19.04 | | | | |
| | | r<=2 | 6.60 | | | | |
| Spain | 2 | r=0 | 131.40 | 2 | 1 | 1 | 1 |
| | | r<=1 | 19.21 | | | | |
| | | r<=2 | 6.74 | | | | |
| Sweden | 3 | r=0 | 84.56 | 1 | 1 | 1 | 1 |
| | | r<=1 | 16.22 | | | | |
| | | r<=2 | 2.87 | | | | |
| United Kingdom | 4 | r=0 | 74.07 | 1 | 1 | 1 | 1 |
| | | r<=1 | 19.05 | | | | |
| | | r<=2 | 6.22 | | | | |

The underlying vector autoregressions included 12 seasonal dummies and different lags.

Lag: number of lags chosen based on AIC & Cauchy limits of residuals.

For α=1.5, the Caner' critical values for a test of size 5% are 34.99, 19.90 and 8.74 for r=0, r=1 and r=2 respectively

For α=1.5, the Caner' critical values for a test of size 2.5% are 40.71, 24.22 and 11.34 for r=0, r=1 and r=2 respectively

For α=1.1, the Caner' critical values for a test of size 5% are 42.57, 23.23 and 9.50 for r=0, r=1 and r=2 respectively

For α=1.1, the Caner' critical values for a test of size 2.5% are 53.88, 31.35 and 13.36 for r=0, r=1 and r=2 respectively

**Table 5.11 Maximum eigenvalue test statistics for the hypothesis about the number of cointegrating vectors in PPP (WPI as price level)**

| | | | Gaussian | | Non-Gaussian | |
|---|---|---|---|---|---|---|
| **Country** | **$H_0$** | **Maximum eigenvalue** | **Rank(5%)** | **Rank(2.5%)** | **Rank(5%)** | **Rank(2.5%)** |
| Japan | r=0 | 18.38 | 0 | 0 | 0 | 0 |
| | r=1 | 9.09 | | | | |
| | r=2 | 5.10 | | | | |
| United Kingdom | r=0 | 39.70 | 3 | 3 | 3 | 1 |
| | r=1 | 17.22 | | | | |
| | r=2 | 10.57 | | | | |

This table corresponds to table 5.5

**Table 5.12 Trace test statistics for the hypothesis about the number of cointegrating vectors in PPP (WPI as price level)**

| | | | Gaussian | | Non-Gaussian | |
|---|---|---|---|---|---|---|
| **Country** | **$H_0$** | **Trace** | **Rank(5%)** | **Rank(2.5%)** | **Rank(5%)** | **Rank(2.5%)** |
| Japan | r=0 | 32.57 | 1 | 0 | 0 | 0 |
| | r<=1 | 14.19 | | | | |
| | r<=2 | 5.10 | | | | |
| United Kingdom | r=0 | 67.49 | 3 | 3 | 3 | 2 |
| | r<=1 | 27.79 | | | | |
| | r<=2 | 10.57 | | | | |

This table corresponds to table 5.6

**Table 5.13 Maximum eigenvalue test statistics for Johansen's multivariate cointegration tests in PPP (WPI as price level)**

| **Country** | **Lag** | $H_0$ | **Maximum eigenvalue** | **Rank(5%)** | **Rank(2.5%)** |
|---|---|---|---|---|---|
| Japan | 10 | r=0 | 22.86 | 1 | 0 |
| | | r=1 | 13.58 | | |
| | | r=2 | 6.08 | | |
| United Kingdom | 11 | r=0 | 45.31 | 2 | 2 |
| | | r=1 | 20.61 | | |
| | | r=2 | 7.15 | | |

This table corresponds to table 5.7

**Table 5.14 Trace test statistics for Johansen's multivariate cointegration tests in PPP (WPI as price level)**

| Country | Lag | $H_0$ | Trace | Rank(5%) | Rank(2.5%) |
|---|---|---|---|---|---|
| Japan | 10 | r=0 | 43.20 | 2 | 1 |
| | | r<=1 | 14.57 | | |
| | | r<=2 | 6.33 | | |
| United Kingdom | 11 | r=0 | 73.07 | 2 | 2 |
| | | r<=1 | 27.77 | | |
| | | r<=2 | 7.15 | | |

This table corresponds to table 5.8

**Table 5.15 Maximum eigenvalue test statistics for Caner's multivariate cointegration tests in PPP (WPI as price level)**

| | | | Maximum | α=1.5 | | α=1.1 | |
|---|---|---|---|---|---|---|---|
| Country | Lag | $H_0$ | Eigenvalue | Rank(5%) | Rank(2.5%) | Rank(5%) | Rank(2.5%) |
| Japan | 10 | r=0 | 22.86 | 0 | 0 | 0 | 0 |
| | | r=1 | 13.58 | | | | |
| | | r=2 | 6.08 | | | | |
| United Kingdom | 11 | r=0 | 45.31 | 2 | 1 | 2 | 1 |
| | | r=1 | 20.61 | | | | |
| | | r=2 | 7.15 | | | | |

This table corresponds to table 5.9

**Table 5.16 Trace test statistics for Caner's multivariate cointegration tests in PPP (WPI as price level)**

| | | | | α=1.5 | | α=1.1 | |
|---|---|---|---|---|---|---|---|
| Country | Lag | $H_0$ | Trace | Rank(5%) | Rank(2.5%) | Rank(5%) | Rank(2.5%) |
| Japan | 10 | r=0 | 43.20 | 1 | 1 | 1 | 0 |
| | | r<=1 | 14.57 | | | | |
| | | r<=2 | 6.33 | | | | |
| United Kingdom | 11 | r=0 | 73.07 | 2 | 2 | 2 | 2 |
| | | r<=1 | 27.77 | | | | |
| | | r<=2 | 7.15 | | | | |

This table corresponds to table 5.10

**Table 5.17 Restriction tests (univariate model) in PPP**

| Price Level | Country | Lag | Maximum eigenvalue | Trace |
|---|---|---|---|---|
| CPI | Belgium | 2 | 3.32 | 3.32 |
| | Canada | 12 | 3.97 | 3.97 |
| | Denmark | 2 | 4.12 | 4.12 |
| | France | 4 | 4.99 | 4.99 |
| | Germany | 2 | 4.28 | 4.28 |
| | Italy | 3 | 4.16 | 4.16 |
| | Japan | 4 | 3.74 | 3.74 |
| | Netherlands | 2 | 4.36 | 4.36 |
| | Norway | 2 | 5.93 | 5.93 |
| | Spain | 2 | 3.06 | 3.06 |
| | Sweden | 3 | 2.54 | 2.54 |
| | United Kingdom | 4 | 6.03 | 6.03 |
| WPI | Japan | 12 | 4.24 | 4.24 |
| | United Kingdom | 3 | 2.87 | 2.87 |

* =rejecting the null hypothesis of no cointegrating vector at 5% significant level when $\alpha$=1.1 & 1.5

** =rejecting the null hypothesis of no cointegrating vector at 2.5% significant level when $\alpha$=1.1 & 1.5

For a test of size 5%, Caner's critical values for n=1 is 8.74 when $\alpha$=1.5, and 9.50 when $\alpha$=1.1

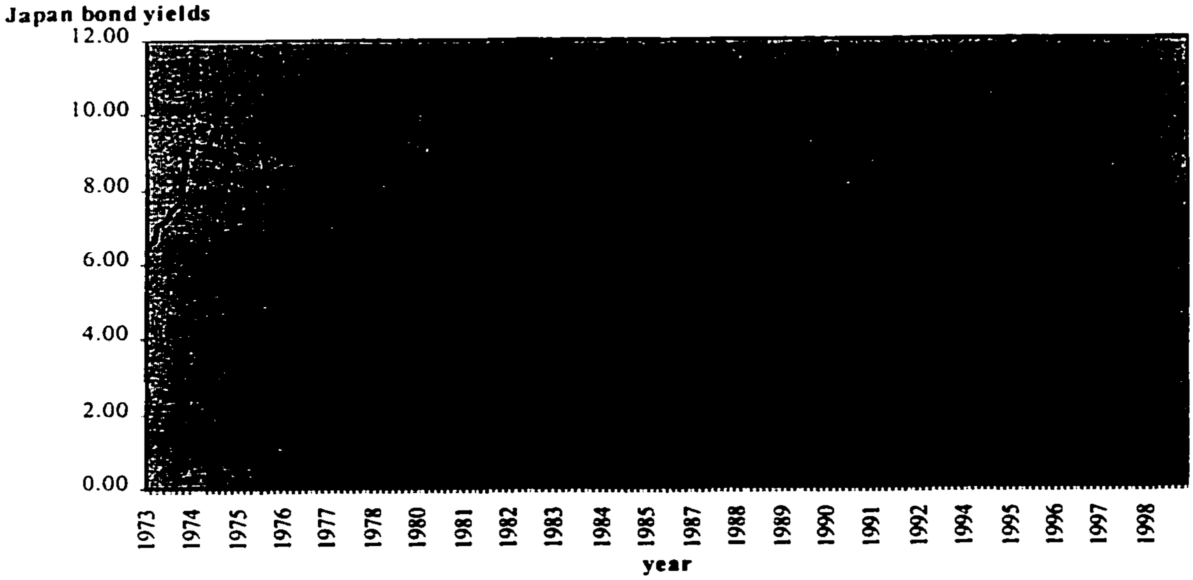
For a test of size 2.5%, Caner's critical values for n=1 is 11.34 when $\alpha$=1.5, and 13.36 when $\alpha$=1.1

Lag: the number of lag in the underlying VAR model, which is decided by Cauchy limits & AIC

# CHAPTER 6
# DATA AND EMPIRICAL RESULTS OF TESTING UIP

## Data

From International Financial Statistics (IFS), we obtained monthly observations running from January 1973 to October 1999. The exchange rates used are all bilateral U.S. dollar rates of the Belgium franc, Canada dollar, France franc, Germany mark, Italy lira, Japan yen, Netherlands guilder, Spain peseta, and U.K. pound. All these data are from line RF except the U.K, which is in line RH. For the nominal interest rate series, we can choose from short-term rates, such as Treasury bill rates and one-year bond yields, and long-term rates, such as ten-year bond yields, for the purpose of comparison. Due to data limitations, the nominal interest rates, which are expressed as $i_t/100$, are from different types of bonds for different countries. The data consist one-year bond yields for Belgium, France, Germany, Japan, Netherlands, Spain and United Kingdom; treasury bill rates for Canada, France, Italy and United Kingdom; and 10-year bond yields for Canada, Italy and United Kingdom. Figures 6.1 – 6.2 display the time paths of bond yields and bond yield logarithmic changes for Japan and United Kingdom between 1973 and 1999. Like the exchange rate and CPI figures shown in chapter 5, informally we see some extreme outlier behavior from the logarithmic change figures for nominal interest rates. Thus, like nominal exchange rates, they may have fat-tail distributions.



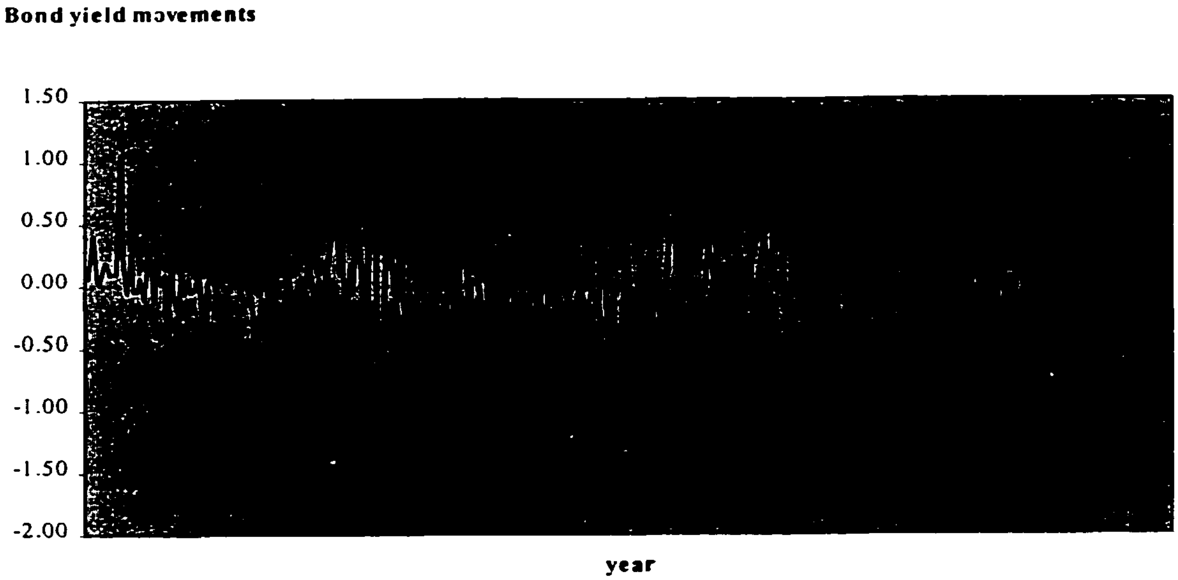
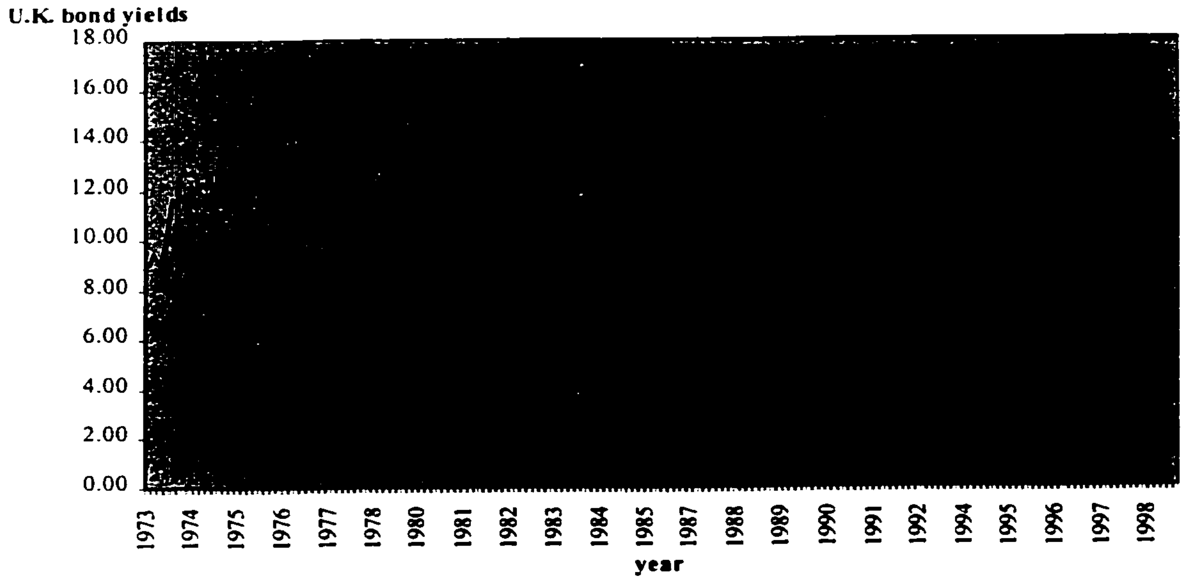




Figure 6.1. The Japan bond yields and bond yield movements, January 1973 – October 1999.



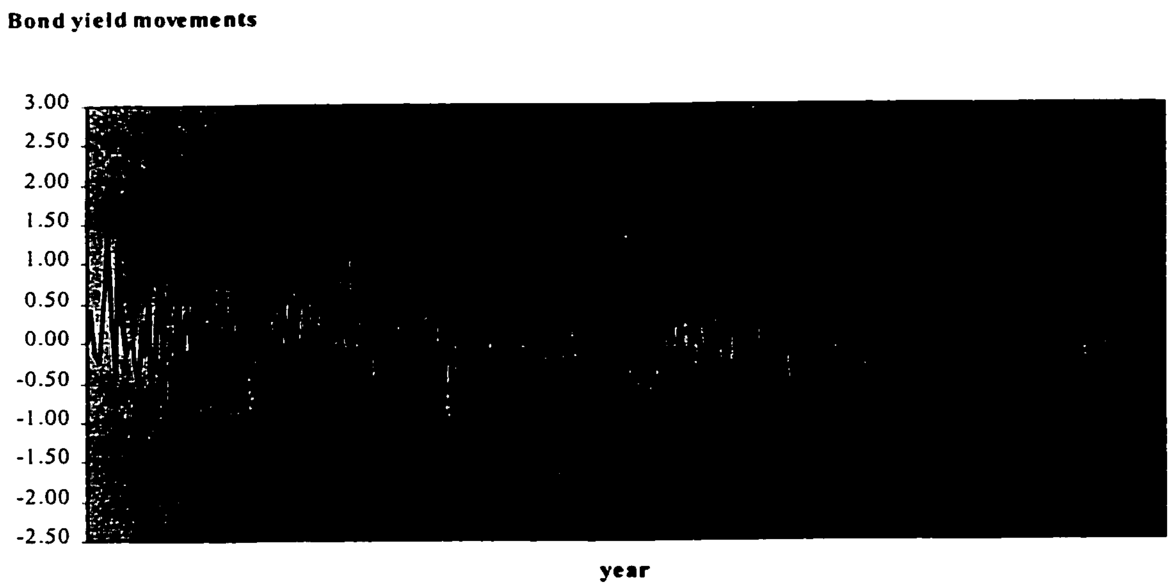




Figure 6.2. The U.K. bond yields and bond yield movements, January 1973 – October 1999.

### Unit Root Tests

Using the critical values in Caner (1998), Phillips-Perron unit root tests are implemented for the nominal interest rate series with the assumption of stable errors. In Table 6.1 the estimated Phillips-Perron statistics are presented for the levels and first differences of the nominal interest rates. We compare the test statistics with Caner's critical values for stability index $\alpha$ being equal to 1 and 1.5. Under the assumption of $\alpha=1$, all the nominal interest rate series except the U.K. long-term bond yield appear to have a single unit root in that although we cannot reject the null of a unit root when these variables appear in levels, we can reject the null when the variables enter as first differences. Deterministic time trends do not appear to be important for these variables except for the U.K. long-term bond yield. Moreover, under the assumption of $\alpha=1.5$, all the nominal interest rate series present the evidence of having a single unit root with or without deterministic time trends. As we mentioned in the previous chapter, even though our exchange rate series and nominal interest rate series may be integrated of different orders, it is still possible for these variables to interact in such a way as to produce a stationary series, i.e. an I(0) series.

### Estimation of Stability Index

As in chapter 5, we implement Box-Jenkins methods using Cauchy based limits instead of Gaussian based limits for model identification and diagnostic checking of i.i.d. residuals. The models selected for the nominal interest rates are provided in Table 6.2. When the first difference of interest rate are already i.i.d. series, the column of fitted model is left blank.

We use the i.i.d. error series of the fitted model to obtain the maximum likelihood estimates and their 95% and 90% confidence intervals of the stability parameters by running the program STABLE in Nolan (1997). The summary of results is contained in Table 6.2. The results show that the estimates of stability index $\alpha$ for Treasury bill rate movements ranged from 1.16 to 1.30, for short-term bond yield movements from 1.37 to 2.00, and for long-term bond yield movements from 1.46 to 1.58. All except Germany bond yield movements have estimates of stability index $\alpha$ less than 2. Also, their 95% and 90% confidence intervals do not include 2. That is, like the exchange rates shown in chapter 5, the assumption of stable errors, i.e. infinite variance errors, may be more appropriate for modeling these nominal interest rate series.

## Residual-based Cointegration Tests

For testing the long-run UIP relationship, residual-based cointegration tests are implemented by estimating equation (3.10) and applying 5% Caner's critical values for stability index $\alpha$=1.5 and 1. Table 6.3 presents the Phillips-Perron test statistics for the residual-based cointegration tests with and without deterministic time trend. We cannot reject the null hypothesis of a unit root, i.e. no cointegration relationship among those variables for either $\alpha$=1.5 or 1, in any case. This finding is the same as the results shown by conventional residual-based cointegration tests with Engle-Yoo's critical values. Moreover, short-term or long-term rates seem not to be important here, because both cases cannot reject the null of no cointegration. Thus, evidence favourable to long-run UIP cannot be found.

**Table 6.1. Phillips-Perron test statistics for a unit root test in interest rates.**

| | | Level | | First Difference |
|---|---|---|---|---|
| **Interest Rate** | **Country** | **No Trend** | **Trend** | **No Trend** |
| Treasury bill | Canada | -2.17663 | -2.81443 | -13.77355* |
| | France | -1.57249 | -2.55015 | -13.58156* |
| | Italy | -0.73444 | -1.71285 | -16.71966* |
| | United Kingdom | -2.03705 | -2.76586 | -12.03937* |
| | United States | -2.17941 | -2.7215 | -13.33113* |
| Short-term Bond | Belgium | -0.83013 | -1.89249 | -14.93978* |
| | France | -0.55760 | -1.78650 | -13.86603* |
| | Germany | -1.66646 | -2.24423 | -11.12387* |
| | Japan | -0.42079 | -2.96731 | -15.14417* |
| | Netherlands | -1.56908 | -2.70014 | -13.41062* |
| | Spain | -0.53461 | -2.45554 | -13.45807* |
| | United Kingdom | -1.90253 | -3.70226 | -12.39745* |
| | United States | -2.11424 | -2.44438 | -11.46469* |
| Long-term Bond | Canada | -1.52716 | -2.25437 | -16.50804* |
| | Italy | -0.99185 | -2.03870 | -12.76270* |
| | United Kingdom | -0.86845 | -4.08940+ | -12.30566* |
| | United States | -1.55413 | -2.17673 | -13.39941* |

Monthly interest rates from IMF

* =rejecting the null hypothesis of a unit root at 5% significant level when $\alpha=1$ & 1.5

+ =rejecting the null hypothesis of a unit root at 5% significant level when $\alpha=1.5$ but not when $\alpha=1$

The 5% Caner's critical values for no trend and trend are, respectively, -3.40 and –3.84 when $\alpha=1.5$

The 5% Caner's critical values for no trend and trend are, respectively, -3.95 and -4.23 when $\alpha=1$

**Table 6.2. Stability index (α) estimates and confidence intervals**

| Interest Rate | Country | Fitted Model* | α | 95% C.I. | 90% C.I. |
|---|---|---|---|---|---|
| Treasury bill | Canada | MA(2) | 1.2816 | ±0.1691 | ±0.1419 |
| | France | AR(1) | 1.2910 | ±0.1721 | ±0.1444 |
| | Italy | | 1.3077 | ±0.1858 | ±0.1559 |
| | United Kingdom | AR(1) | 1.1653 | ±0.1623 | ±0.1362 |
| | United States | MA(1) | 1.1654 | ±0.1599 | ±0.1342 |
| Short-term Bond | Belgium | | 1.7536 | ±0.1042 | ±0.0875 |
| | France | MA(1) | 1.5836 | ±0.1677 | ±0.1407 |
| | Germany | AR(1) | 2.0000 | | |
| | Japan | | 1.4315 | ±0.1719 | ±0.1443 |
| | Netherlands | AR(1) | 1.7373 | ±0.1694 | ±0.1421 |
| | Spain | | 1.8797 | ±0.1096 | ±0.0920 |
| | United Kingdom | MA(1) | 1.3953 | ±0.1725 | ±0.1448 |
| | United States | MA(1) | 1.3758 | ±0.1761 | ±0.1478 |
| Long-term Bond | Canada | | 1.5812 | ±0.1672 | ±0.1403 |
| | Italy | AR(1) | 1.4634 | ±0.1731 | ±0.1453 |
| | United Kingdom | AR(2) | 1.5170 | ±0.1345 | ±0.1129 |
| | United States | | 1.5880 | ±0.1688 | ±0.1417 |

* The blank entries in this column correspond to cases where the series are serially uncorrelated.
C.I. : confidence interval

**Table 6.3. Residual-based cointegration tests in UIP**

| | | P-P | |
|---|---|---|---|
| **Interest Rate** | **Country** | **No Trend** | **Trend** |
| Treasury bill | Canada | -3.28161 | -3.21807 |
| | France | -3.01188 | -3.01056 |
| | Italy | -2.80281 | -2.82574 |
| | Spain | -2.86473 | -2.85397 |
| | United Kingdom | -3.49919 | -3.51067 |
| Short-term Bond | Belgium | -2.93325 | -2.92734 |
| | France | -2.79211 | -2.78848 |
| | Germany | -3.14031 | -3.10834 |
| | Japan | -3.53166 | -3.59485 |
| | Netherlands | -3.14791 | -3.13800 |
| | Spain | -2.78620 | -2.75803 |
| | United Kingdom | -3.30430 | -3.30139 |
| Long-term Bond | Canada | -3.38760 | -3.34235 |
| | Italy | -3.23604 | -3.31872 |
| | United Kingdom | -3.43939 | -3.43144 |

* : rejecting the null hypothesis of no cointegration ( i.e. a unit root) at 5% significant level when $\alpha=1.5$ & 1

+ : rejecting the null hypothesis of no cointegration ( i.e. a unit root) at 5% significant level when $\alpha=1.5$ but not when $\alpha=1$

P-P=Phillips-Perron test statistic

The 5% Engle-Yoo's critical values : -3.93

The 5% Caner's critical values for no trend and trend are, respectively, -4.23 and -4.52 when $\alpha=1.5$

The 5% Caner's critical values for no trend and trend are, respectively, -4.93 and -5.28 when $\alpha=1$

**Multivariate Cointegration Tests**

Under the assumption of normal errors, we could not find a lag length k for the trivariate VAR model, which has the ACF and PACF of its residual series entirely within the Gaussian-based limits. That is, we cannot satisfy the error orthogonality condition in all cases. This finding is consistent with previous research. Alternatively, we use first-order lag and twelfth-order lag in the Gaussian case. Tables 6.4 and 6.5 report our estimates of the maximum eigenvalue and trace statistics (equations 3.7 and 3.6), respectively, along with the critical values from Osterwalk-Lenum (1992) for a test of size 5% and 2.5%. On the basis of the maximum eigenvalue statistics, all the ranks, i.e. the number of cointegrating vectors are equal to or greater than one, and smaller than three. On the basis of the trace statistics, all except Belgium have ranks of one or two. In the case of Belgium, we find three cointegrating vectors when the lag length is one, which implies the variables are stationary, which is not consistent with the results of unit root tests. However, due to the lack of orthogonality, the above evidence may not be strong support for long-run UIP relationship with United States.

On the other hand, under the assumption of stable errors we were able to select VAR lag lengths satisfying the orthogonality condition. The maximum eigenvalue and trace statistics are presented in Tables 6.6 and 6.7, respectively, along with the lag length chosen for the underlying VAR model, and the ranks, i.e. the number of cointegrating vector, decided by Caner's critical values. On the basis of either maximum eigenvalue or trace statistics, all the cases have at least one cointegrating vector for the assumption of $\alpha=1.5$.

For a test of size 5%, whether $\alpha$=1.5 or 1, there is evidence of cointegration in all cases. For a test of size 2.5%, if $\alpha$=1.5 there is at least one cointegrating vector in every case. But if $\alpha$=1 four combinations (Italy (treasury bill), Spain (bond), United Kingdom (bond), and United Kingdom (long-term bond)) appear to have no cointegrating vector. That is, the results of testing UIP relationship are sensitive to stability index $\alpha$ for a test of size 2.5%, not for a test of size 5%.

From the above analysis, we may conclude that the results are consistent and more strongly supportive of long-run UIP relationship with United States under the assumption of stable errors.

For the strict UIP relationship, the symmetry and proportionality conditions, i.e. the coefficient restrictions ($\alpha_0$, $\alpha_1$, $\alpha_2$)=(0, 1, -1) in equation (3.10), need to be tested. By imposing the symmetry and proportionality conditions we reduce the trivariate model for $(s_{t+1}, s_t, i_t - i_t^*)$ to the univariate one for $(s_{t+1} - s_t - (i_t - i_t^*))$. For the univariate model, the maximum eigenvalue and trace statistics are equivalent. In Table 6.8 we include the test statistics, the lag length chosen for the underlying AR model, and Caner's critical values for tests of size 5% and 2.5%. We cannot reject the null hypothesis of no cointegration in any case except Canada with Treasury bill rate, and United Kingdom with short-term rate or long-term rate. That is we cannot find supportive evidence for strict UIP relationships with the United States. The symmetry or proportionality restriction is violated due to finding non-stationarity in the restricted univariate model. The violation of strict UIP may reflect the value of the risk premium as well as other factors such as transaction costs.

**Table 6.4. Johansen's multivariate cointegration tests in UIP**

| Interest Rate | Country | $H_0$ | 1 lag M-eigen | 1 lag Rank (5%) | 1 lag Rank (2.5%) | 12 lag M-eigen | 12 lag Rank (5%) | 12 lag Rank (2.5%) |
|---|---|---|---|---|---|---|---|---|
| Treasury bill | Canada | r=0 | 74.35 | 1 | 1 | 8404.07 | 2 | 1 |
| | | r=1 | 12.04 | | | 15.23 | | |
| | | r=2 | 4.19 | | | 5.0 | | |
| | France | r=0 | 46.73 | 1 | 1 | 7811.30 | 1 | 1 |
| | | r=1 | 9.31 | | | 8.96 | | |
| | | r=2 | 4.96 | | | 6.18 | | |
| | Italy | r=0 | 51.06 | 1 | 1 | 7965.99 | 1 | 1 |
| | | r=1 | 10.39 | | | 11.70 | | |
| | | r=2 | 7.00 | | | 8.09 | | |
| | United Kingdom | r=0 | 43.78 | 1 | 1 | 5639.88 | 1 | 1 |
| | | r=1 | 13.43 | | | 8.55 | | |
| | | r=2 | 6.69 | | | 3.92 | | |
| Short-term Bond | Belgium | r=0 | 60.88 | 1 | 1 | (11 lag)314.24 | 1 | 1 |
| | | r=1 | 13.48 | | | 10.08 | | |
| | | r=2 | 9.84 | | | 5.47 | | |
| | France | r=0 | 55.03 | 1 | 1 | 7714.98 | 1 | 1 |
| | | r=1 | 11.98 | | | 8.99 | | |
| | | r=2 | 6.41 | | | 5.26 | | |
| | Germany | r=0 | 50.78 | 1 | 1 | (11 lag)297.45 | 1 | 1 |
| | | r=1 | 8.75 | | | 9.76 | | |
| | | r=2 | 6.88 | | | 2.58 | | |
| | Japan | r=0 | 52.73 | 1 | 1 | (11 lag)337.33 | 1 | 1 |
| | | r=1 | 9.55 | | | 8.71 | | |
| | | r=2 | 3.68 | | | 2.59 | | |
| | Netherlands | r=0 | 50.10 | 1 | 1 | (11 lag)309.27 | 1 | 1 |
| | | r=1 | 8.69 | | | 9.07 | | |
| | | r=2 | 8.05 | | | 3.28 | | |
| | Spain | r=0 | 29.78 | 2 | 2 | 5460.88 | 2 | 1 |
| | | r=1 | 23.38 | | | 15.41 | | |
| | | r=2 | 5.05 | | | 2.26 | | |
| | United Kingdom | r=0 | 39.42 | 1 | 1 | 7603.48 | 1 | 1 |
| | | r=1 | 11.61 | | | 12.47 | | |
| | | r=2 | 8.02 | | | 5.31 | | |
| Long-term Bond | Canada | r=0 | 73.90 | 2 | 2 | 8306.42 | 1 | 1 |
| | | r=1 | 23.57 | | | 7.92 | | |
| | | r=2 | 4.69 | | | 5.90 | | |
| | Italy | r=0 | 64.63 | 1 | 1 | 6709.91 | 1 | 1 |
| | | r=1 | 8.16 | | | 12.07 | | |
| | | r=2 | 4.73 | | | 5.98 | | |
| | United Kingdom | r=0 | 41.57 | 1 | 1 | 7969.98 | 2 | 2 |
| | | r=1 | 11.94 | | | 18.80 | | |
| | | r=2 | 3.43 | | | 4.14 | | |

M-eigen: maximum eigenvalue statistics r : ranks (i.e. number of cointegrating vectors)

The critical values for a test of size 5% are 21.07, 14.90 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are 22.89, 17.07 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).

**Table 6.5. Johansen's multivariate cointegration tests in UIP**

| Interest Rate | Country | $H_0$ | 1 lag Trace | 1 lag Rank (5%) | 1 lag Rank (2.5%) | 12 lag Trace | 12 lag Rank (5%) | 12 lag Rank (2.5%) |
|---|---|---|---|---|---|---|---|---|
| Treasury bill | Canada | r=0 | 91.57 | 1 | 1 | 8424.37 | 2 | 2 |
| | | r<=1 | 16.23 | | | 20.30 | | |
| | | r<=2 | 4.19 | | | 5.07 | | |
| | France | r=0 | 61.00 | 1 | 1 | 7826.40 | 1 | 1 |
| | | r<=1 | 14.27 | | | 15.10 | | |
| | | r<=2 | 4.96 | | | 6.18 | | |
| | Italy | r=0 | 68.45 | 1 | 1 | 5639.88 | 1 | 1 |
| | | r<=1 | 17.39 | | | 8.55 | | |
| | | r<=2 | 7.00 | | | 3.92 | | |
| | United Kingdom | r=0 | 63.91 | 2 | 2 | 7965.99 | 1 | 1 |
| | | r<=1 | 20.13 | | | 11.70 | | |
| | | r<=2 | 6.69 | | | 8.09 | | |
| Short-term Bond | Belgium | r=0 | 84.20 | 3 | 3 | (11 lag)329.79 | 1 | 1 |
| | | r<=1 | 23.32 | | | 15.55 | | |
| | | r<=2 | 9.84 | | | 5.47 | | |
| | France | r=0 | 73.42 | 2 | 1 | 7729.24 | 1 | 1 |
| | | r<=1 | 18.39 | | | 14.26 | | |
| | | r<=2 | 6.41 | | | 5.26 | | |
| | Germany | r=0 | 66.41 | 1 | 1 | (11 lag)309.78 | 1 | 1 |
| | | r<=1 | 15.63 | | | 12.34 | | |
| | | r<=2 | 6.88 | | | 2.58 | | |
| | Japan | r=0 | 65.96 | 1 | 1 | (11 lag)348.63 | 1 | 1 |
| | | r<=1 | 13.22 | | | 11.30 | | |
| | | r<=2 | 3.68 | | | 2.59 | | |
| | Netherlands | r=0 | 66.83 | 1 | 1 | (11 lag)321.63 | 1 | 1 |
| | | r<=1 | 16.73 | | | 12.35 | | |
| | | r<=2 | 8.05 | | | 3.28 | | |
| | Spain | r=0 | 58.20 | 2 | 2 | 5478.55 | 1 | 1 |
| | | r<=1 | 28.42 | | | 17.67 | | |
| | | r<=2 | 5.05 | | | 2.26 | | |
| | United Kingdom | r=0 | 59.05 | 2 | 1 | 7621.27 | 1 | 1 |
| | | r<=1 | 19.63 | | | 17.79 | | |
| | | r<=2 | 8.02 | | | 5.31 | | |
| Long-term Bond | Canada | r=0 | 102.17 | 2 | 2 | 8320.25 | 1 | 1 |
| | | r<=1 | 28.26 | | | 13.83 | | |
| | | r<=2 | 4.69 | | | 5.90 | | |
| | Italy | r=0 | 77.52 | 1 | 1 | 6727.95 | 2 | 1 |
| | | r<=1 | 12.89 | | | 18.05 | | |
| | | r<=2 | 4.73 | | | 5.98 | | |
| | United Kingdom | r=0 | 56.94 | 1 | 1 | 7992.91 | 2 | 2 |
| | | r<=1 | 15.37 | | | 22.93 | | |
| | | r<=2 | 3.43 | | | 4.14 | | |

Trace : trace statistics

The critical values for a test of size 5% are 31.52, 17.95 and 8.18 for r=0, r=1 and r=2 respectively, ones for a test of size 2.5% are, 34.48, 20.08 and 9.72 for r=0, r=1 and r=2 respectively, from Osterwald-Lenum (1992).

**Table 6.6. Caner's multivariate cointegration tests in UIP**

| | | | | | α=1.5 | | α=1.1 | |
|---|---|---|---|---|---|---|---|---|
| **Interest Rate** | **Country** | **$H_0$** | **Lag** | **M-eigen** | **Rank(5%)** | **Rank(2.5%)** | **Rank(5%)** | **Rank(2.5%)** |
| Treasury Bill | Canada | r=0 | 8 | 87.59 | 1 | 1 | 1 | 1 |
| | | r=1 | | 16.16 | | | | |
| | | r=2 | | 4.63 | | | | |
| | France | r=0 | 7 | 77.27 | 1 | 1 | 1 | 1 |
| | | r=1 | | 8.13 | | | | |
| | | r=2 | | 3.30 | | | | |
| | Italy | r=0 | 2 | 33.56 | 1 | 1 | 1 | 0 |
| | | r=1 | | 10.74 | | | | |
| | | r=2 | | 5.17 | | | | |
| | United Kingdom | r=0 | 3 | 43.57 | 1 | 1 | 1 | 1 |
| | | r=1 | | 14.36 | | | | |
| | | r=2 | | 6.15 | | | | |
| Short-term Bond | Belgium | r=0 | 7 | 65.08 | 1 | 1 | 1 | 1 |
| | | r=1 | | 7.46 | | | | |
| | | r=2 | | 4.48 | | | | |
| | France | r=0 | 8 | 83.87 | 1 | 1 | 1 | 1 |
| | | r=1 | | 5.76 | | | | |
| | | r=2 | | 3.79 | | | | |
| | Germany | r=0 | 8 | 78.37 | 1 | 1 | 1 | 1 |
| | | r=1 | | 9.34 | | | | |
| | | r=2 | | 2.06 | | | | |
| | Japan | r=0 | 7 | 75.10 | 1 | 1 | 1 | 1 |
| | | r=1 | | 8.87 | | | | |
| | | r=2 | | 2.31 | | | | |
| | Netherlands | r=0 | 8 | 80.29 | 1 | 1 | 1 | 1 |
| | | r=1 | | 7.97 | | | | |
| | | r=2 | | 2.64 | | | | |
| | Spain | r=0 | 3 | 34.57 | 2 | 2 | 2 | 0 |
| | | r=1 | | 20.93 | | | | |
| | | r=2 | | 4.87 | | | | |
| | United Kingdom | r=0 | 3 | 41.12 | 1 | 1 | 1 | 0 |
| | | r=1 | | 14.13 | | | | |
| | | r=2 | | 5.96 | | | | |
| Long-term Bond | Canada | r=0 | 2 | 58.23 | 2 | 1 | 1 | 1 |
| | | r=1 | | 17.09 | | | | |
| | | r=2 | | 4.82 | | | | |
| | Italy | r=0 | 7 | 72.59 | 1 | 1 | 1 | 1 |
| | | r=1 | | 12.55 | | | | |
| | | r=2 | | 8.94 | | | | |
| | United Kingdom | r=0 | 4 | 40.23 | 1 | 1 | 1 | 0 |
| | | r=1 | | 11.67 | | | | |
| | | r=2 | | 3.26 | | | | |

For α=1.5, the Caner' critical values for a test of size 5% are 24.50, 16.82 and 8.74 for r=0, r=1 and r=2 respectively
For α=1.5, the Caner' critical values for a test of size 2.5% are 29.77, 20.66 and 11.34 for r=0, r=1 and r=2 respectively
For α=1.1, the Caner' critical values for a test of size 5% are 32.05, 20.27 and 9.50 for r=0, r=1 and r=2 respectively
For α=1.1, the Caner' critical values for a test of size 2.5% are 42.96, 27.72 and 13.36 for r=0, r=1 and r=2 respectively

**Table 6.7. Caner's multivariate cointegration tests in UIP**

| Interest Rate | Country | $H_0$ | Lag | Trace | α=1.5 Rank(5%) | α=1.5 Rank(2.5%) | α=1.1 Rank(5%) | α=1.1 Rank(2.5%) |
|---|---|---|---|---|---|---|---|---|
| Treasury bill | Canada | R=0 | 8 | 108.37 | 2 | 1 | 1 | 1 |
| | | R<=1 | | 20.79 | | | | |
| | | r<=2 | | 4.36 | | | | |
| | France | r=0 | 7 | 88.70 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 11.43 | | | | |
| | | r<=2 | | 3.30 | | | | |
| | Italy | r=0 | 2 | 49.47 | 1 | 1 | 1 | 0 |
| | | r<=1 | | 15.91 | | | | |
| | | r<=2 | | 5.17 | | | | |
| | United Kingdom | r=0 | 3 | 64.08 | 2 | 1 | 1 | 1 |
| | | r<=1 | | 20.51 | | | | |
| | | r<=2 | | 6.15 | | | | |
| Short-term Bond | Belgium | r=0 | 7 | 77.02 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 11.94 | | | | |
| | | r<=2 | | 4.48 | | | | |
| | France | r=0 | 8 | 93.42 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 9.55 | | | | |
| | | r<=2 | | 3.79 | | | | |
| | Germany | r=0 | 8 | 89.17 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 11.40 | | | | |
| | | r<=2 | | 2.06 | | | | |
| | Japan | r=0 | 7 | 86.28 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 11.18 | | | | |
| | | r<=2 | | 2.31 | | | | |
| | Netherlands | r=0 | 8 | 90.91 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 10.62 | | | | |
| | | r<=2 | | 2.64 | | | | |
| | Spain | r=0 | 3 | 60.38 | 2 | 2 | 2 | 1 |
| | | r<=1 | | 25.81 | | | | |
| | | r<=2 | | 4.87 | | | | |
| | United Kingdom | r=0 | 3 | 61.21 | 2 | 1 | 1 | 1 |
| | | r<=1 | | 20.08 | | | | |
| | | r<=2 | | 5.96 | | | | |
| Long-term Bond | Canada | r=0 | 2 | 80.13 | 2 | 1 | 1 | 1 |
| | | r<=1 | | 21.90 | | | | |
| | | r<=2 | | 4.82 | | | | |
| | Italy | r=0 | 7 | 94.09 | 3 | 1 | 1 | 1 |
| | | r<=1 | | 21.50 | | | | |
| | | r<=2 | | 8.94 | | | | |
| | United Kingdom | r=0 | 4 | 55.15 | 1 | 1 | 1 | 1 |
| | | r<=1 | | 14.93 | | | | |
| | | r<=2 | | 3.26 | | | | |

For α=1.5, the Caner' critical values for a test of size 5% are 34.99, 19.90 and 8.74 for r=0, r=1 and r=2 respectively
For α=1.5, the Caner' critical values for a test of size 2.5% are 40.71, 24.22 and 11.34 for r=0, r=1 and r=2 respectively
For α=1.1, the Caner' critical values for a test of size 5% are 42.57, 23.23 and 9.50 for r=0, r=1 and r=2 respectively
For α=1.1, the Caner' critical values for a test of size 2.5% are 53.88, 31.35 and 13.36 for r=0, r=1 and r=2 respectively

**Table 6.8. Restriction tests (Univariate model) in UIP**

| Interest Rate | Country | Lag | Maximum-eigenvalue | Trace |
|---|---|---|---|---|
| Treasury bill | Canada | 1 | 9.59* | 9.59* |
| | France | 10 | 7.60 | 7.60 |
| | Italy | 9 | 4.52 | 4.52 |
| | United Kingdom | 3 | 12.28* | 12.28* |
| Short-term Bond | Belgium | 10 | 6.90 | 6.90 |
| | France | 10 | 7.34 | 7.34 |
| | Germany | 11 | 3.12 | 3.12 |
| | Japan | 10 | 7.93 | 7.93 |
| | Netherlands | 10 | 4.78 | 4.78 |
| | Spain | 9 | 4.36 | 4.36 |
| | United Kingdom | 3 | 12.79* | 12.79* |
| Long-term Bond | Canada | 3 | 7.65 | 7.65 |
| | Italy | 6 | 4.49 | 4.49 |
| | United Kingdom | 4 | 11.78* | 11.78* |

* =rejecting the null hypothesis of no cointegrating vector at 5% significant level when $\alpha=1.1$ & 1.5

** =rejecting the null hypothesis of no cointegrating vector at 2.5% significant level when $\alpha=1.1$ & 1.5

For a test of size 5%, Caner's critical values for n=1 is 8.74 when $\alpha=1.5$, and 9.50 when $\alpha=1.1$

For a test of size 2.5%, Caner's critical values for n=1 is 11.34 when $\alpha=1.5$, and 13.36 when $\alpha=1.1$

Lag: the number of lag in the underlying VAR model, which is decided by Cauchy limits & AIC

# CHAPTER 7
# SUMMARY AND CONCLUSIONS

In this study, we have re-examined the purchasing power parity and uncovered interest parity concepts using residual-based and multivariate likelihood-based cointegration tests under the assumption of stable errors. Estimating the stability indices of the exchange rate, price index, and nominal interest rate series indicated in the previous chapters, we find evidence that most of them have an index of stability $\alpha$ less than 2. That is, it appears from the evidence that a stable non-Gaussian model may be more appropriate for these series in our data. Phillips-Perron unit root tests, along with the critical values in Caner (1998), implemented for determining the order of integration of those series generally cannot reject the null of a unit root. The finding of the non-Gaussian stable errors and the unit root in those series provide the motivation for re-doing the cointegration tests for the PPP and UIP relationships. Under the assumption of stable non-Gaussian innovations, we implement the residual-based and multivariate likelihood-based cointegration tests of Caner (1998) which extended the Johansen (1988) method from finite variance errors to the infinite variance errors.

For the PPP hypothesis, this study has investigated the relationship between the nominal exchange rate and the domestic and foreign price levels for Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden and United Kingdom against United States over the period 1973—1999. With the stable error assumption the results obtained by the multivariate likelihood-based cointegration tests demonstrated that weak-form PPP receives stronger support from the data, while with the normal error assumption the results show some evidence of supporting the weak-form PPP relationship with United States. By this finding, we may conclude that the stronger supporting evidence is found due to the appropriate assumption of the error series. However, the restrictions for strong-form PPP have been violated. There are ways to explain this finding. First, there may be measurement errors and traded/nontraded biased that generate this result, although it is hard to see that the gross violations observed in the data can be wholly attributed to these kinds of factors. The other interpretation is that there have been real disturbances and capital movements during the recent float which upset the proportionality relationship.

For the UIP hypothesis, the relationship between nominal exchange rate and interet rate difference is investigated. Due to some data limitations, the nominal interest rates are from different types of bonds for some countries. Bond yields for Belgium, France, Germany, Japan, Netherlands, Spain and United Kingdom, treasury bill rates for Canada, France, Italy and United Kingdom, long-term bond yields for Canada, Italy and United Kingdom are included in the data. According to the evidence shown in chapter 6, we may conclude that the results are consistent and strongly supportive of long-run UIP relationship with United States under the assumption of stable errors. However, like PPP, The restrictions for strict UIP relationship are violated. The violation may reflect the value of the risk premium as well as other factors such as transaction costs.

In contrast, using residual-based cointegration tests for either PPP or UIP relationship in the data proves to be a disappointing exercise, since the null hypothesis of a unit root (i.e.

no cointegration) cannot be rejected whether we assume normal errors or stable errors. However, it is widely accepted that these tests have relatively low power against a broad range of stationary alternatives.

Moreover, there is a limitation in this study. In the residual-based and multivariate likelihood-based cointegration tests, we assume all variables in the relationship have the same index of stability. Though the point estimates of $\alpha$ are slightly different among those series, there is no econometric test for equality of the index of stability.

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